Robust Adaptive Controller for a Class of Uncertain Nonlinear Systems with Disturbances

Ngo Tri Nam Cuong, Le Van Chuong, and Mai The Anh

Abstract This paper presents a method to synthesize the controller for uncertain nonlinear systems based on a combination of sliding mode control, adaptive control, and radial basis function (RBF) neural network. We propose an adaptive control law based on the RBF neural network to identify and compensate for variable parameter components, nonlinear function vectors, and external disturbance. The main linear component is built based on a sliding control. The designed controller has the advantage of being resistant to the elements of uncertainty and has a high control quality.

Keywords Nonlinear systems · Adaptive control · System identification

1 Introduction

In practice, the uncertain nonlinear systems are affected by external disturbances which are very common. The existence of uncertain parameters adversely affects the performance of the system. Control design for such a class of objects has attracted the attention of many researchers in past decades. A combination of the adaptive control method and the neural network has been shown in the researches [1–4], in which nonlinear components and external disturbance are identified using the neural network to generate a compensation control signal for the uncertain components. Some researches on adaptive control have been implemented for such variable nonlinear systems, where variable parameters are identified and adjusted by adaptive control law [5–7]. In [8], a control law is built based on sliding mode control in which uncertain components are considered for control design, and thus the designed system is stability. The control design using backstepping and fuzzy techniques is

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implemented in [9], where the external disturbance is identified and compensated by fuzzy logic, the variable parameters are determined based on the backstepping technique. In the papers [10, 11], identifying and correcting uncertain parameters are also implemented based on the backstepping control. Thus, there are many different methods to synthesize control systems for the class of nonlinear objects under the affection of variable parameters and external disturbance. Many results have been shown advantages of diffirent methods, however enhancing performmance of the designed system is still problems of interest to many researches. This paper presents a method of synthesizing a stable adaptive controller based on combining adaptive control, sliding control, and RBF neural network.

2 Problem Formulation

A multi-input multi-output (MIMO) nonlinear system will be considered in the paper:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + [\mathbf{B} + \mathbf{\Delta B}]\mathbf{u} + \mathbf{f}(\mathbf{x}) + \mathbf{d}(t), \tag{1}$$

where $\mathbf{x} = [x_1, x_2 ... x_n]^T$ is state vector; $\mathbf{u} = [u_1, u_2 ... u_m]^T$ is control vector; $\mathbf{A} \in \mathbb{R}^{n\times n}$ is Hurwitz matrix with fixed elements; $\mathbf{B} \in \mathbb{R}^{n\times m}$ is matrix with fixed elements; $\mathbf{\Delta B} \in \mathbb{R}^{n\times m}$ matrix matched uncertainty; $\mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_n(\mathbf{x})]^T$ is smooth nonlinear vetor, matched uncertainty; $\mathbf{d}(t) = [d_1(t), d_2(t), ..., d_n(t)]^T$ is external disturbance vector with slow varible elements, matched uncertainty $|d_i(t)| \leq d_M$.

The block diagram of the designed system using identification structure with compensation of uncertain component and external disturbance is shown in Fig. 1. MODEL is the identification model; IDENT is the identification block; COMP is the compensation block of uncertain components and external disturbance; SMC is the sliding mode controller.

The control signal can be considered as follow:

$$\mathbf{u} = \mathbf{u}_{smc} + \mathbf{u}_c, \tag{2}$$

where $\mathbf{u}_{smc}$ is control signal vector of SMC; $\mathbf{u}_c$ is control signal vector for compensation of uncertain component and external disturbance.

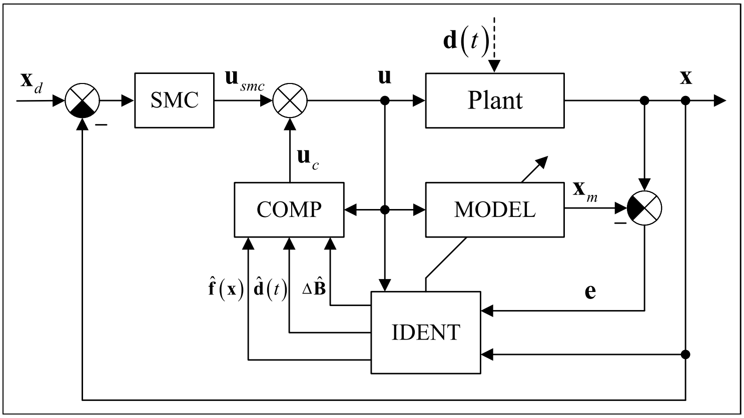

Fig. 1 Block diagram of the designed system

3 Algorithm for Identification and Compensation of Uncertain Parameters

Uncertain components $\mathbf{\Delta B}, \mathbf{f}(\mathbf{x}), \mathbf{d}(t)$ in (1) need to be identified and adjusted for compensation. The identification model for uncertain parameters in (1) can be written:

$$\dot{\mathbf{x}}_m = \mathbf{A}\mathbf{x}_m + [\mathbf{B} + \mathbf{\Delta}\hat{\mathbf{B}}]\mathbf{u} + \hat{\mathbf{f}}(\mathbf{x}) + \hat{\mathbf{d}}(t), \tag{3}$$

where $\mathbf{x}_m = [x_{m1}, x_{m2}...x_{mn}]^T$ is state vector of the model; $\mathbf{\Delta}\hat{\mathbf{B}}$ is the estimated matrix of $\mathbf{\Delta B}$ which is defined by elements Δb_{ij}; $\hat{\mathbf{f}}(\mathbf{x}) = [\hat{f}_1(\mathbf{x}), \hat{f}_2(\mathbf{x}), ..., \hat{f}_n(\mathbf{x})]^T$ is the estimated vector of $\mathbf{f}(\mathbf{x})$; $\hat{\mathbf{d}}(t) = [\hat{d}_1(t), \hat{d}_2(t), ..., \hat{d}_n(t)]^T$ is the estimated vector of $\mathbf{d}(t)$.

From (1) and (3), we have:

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} + \mathbf{\Delta}\tilde{\mathbf{B}}\mathbf{u} + \tilde{\mathbf{f}}(\mathbf{x}) + \tilde{\mathbf{d}}(t), \tag{4}$$

where $\mathbf{e} = \mathbf{x} - \mathbf{x}_m$; $\mathbf{\Delta}\tilde{\mathbf{B}} = \mathbf{\Delta B} - \mathbf{\Delta}\hat{\mathbf{B}}$; $\tilde{\mathbf{f}}(\mathbf{x}) = \mathbf{f}(\mathbf{x}) - \hat{\mathbf{f}}(\mathbf{x})$; $\tilde{\mathbf{d}}(t) = \mathbf{d}(t) - \hat{\mathbf{d}}(t)$. Identification progress will be converging when $\mathbf{\Delta}\tilde{\mathbf{B}} \to 0, \tilde{\mathbf{f}}(\mathbf{x}) \to 0, \tilde{\mathbf{d}}(t) \to 0$. Because $\mathbf{A}$ is defined by a Hurwitz matrix, so $\mathbf{e} \to 0$, and (4) is stability.

With $\mathbf{f}(\mathbf{x})$ is a smooth function vector, by using a RBF neural network for the approximation. The elements of $\mathbf{f}(\mathbf{x})$ can be written:

$$f_i(\mathbf{x}) = \sum_{j=1}^{L} w_{ij}^* \phi_{ij}(\mathbf{x}) + \varepsilon_i, \tag{5}$$

$\forall i = \overline{1,n}$; $j = \overline{1,L}$, where L is number of basis function with a large enough number to guarantee the error $|\varepsilon_i| < \varepsilon_i^m$, $w_{ij}^* = const$ is the ideal weights. The basis functions are selected by the following form:

$$\phi_{ij}(\mathbf{x}) = \exp\left(\frac{\|\mathbf{x} - \mathbf{c}_{ij}\|^2}{2\sigma_{ij}^2}\right), \tag{6}$$

where $\mathbf{c}_{ij}$ are the position of the center of the basis functions $\phi_{ij}(\mathbf{x})$, and σ_{ij} are the standard deviation of the basis functions. The evaluation vector $\hat{\mathbf{f}}(\mathbf{x})$ is defined by (6) with adjusted weights $\hat{w}_{ij}$:

$$\hat{f}_i(\mathbf{x}) = \sum_{j=1}^{L} \hat{w}_{ij}\phi_{ij}(\mathbf{x}), i = \overline{1,n}. \tag{7}$$

Training of the RBF neural network is implemented by adjustment of the weights $\hat{w}_{ij}$ in comparison with the ideal weights w_{ij}^*:

$$\tilde{w}_{ij} = w_{ij}^* - \hat{w}_{ij}, \tag{8}$$

from (5), (7) and (8), we have:

$$f_i(\mathbf{x}) = \hat{f}_i(\mathbf{x}) + \varepsilon_i \rightarrow \tilde{f}(\mathbf{x}) = \sum_{j=1}^{L} \tilde{w}_{ij}\phi_{ij}(\mathbf{x}) + \varepsilon_i, \tag{9}$$

ε_i is the approximate error with a sufficiently small value.

Theorem 1 *Equation (4) are stable when the following conditions are satisfied:*

$$\|\mathbf{e}\| > \frac{2\sum_{i=1}^{n} \varepsilon_i \|\bar{\mathbf{P}}_i\|}{r_{\min}(\mathbf{Q})}; \tag{10}$$

$$\mathbf{u}^T \mathbf{\Delta}\tilde{\mathbf{B}}^T \mathbf{P}\mathbf{e} + \sum_{i=1}^{n}\sum_{j=1}^{m} \Delta\dot{\tilde{b}}_{ij}\Delta\tilde{b}_{ij} = 0; \tag{11}$$

$$\mathbf{e}^T\mathbf{P}\begin{bmatrix} \sum_{j=1}^{L} \tilde{w}_{1j}\phi_{ij}(\mathbf{x}) \\ \vdots \\ \sum_{j=1}^{L} \tilde{w}_{nj}\phi_{ij}(\mathbf{x}) \end{bmatrix} + \sum_{i=1}^{n}\sum_{j=1}^{L} \dot{\tilde{w}}_{ij}\tilde{w}_{ij} = 0; \tag{12}$$

$$\mathbf{e}^T\mathbf{P}\tilde{\mathbf{d}}\,(t) + \sum_{i=1}^{n} \dot{\tilde{d}}_i \tilde{d}_i = 0. \tag{13}$$

P *is a positive definite symmetric matrix* □

Proof. For Eq. (4), the Lyapunov function is selected as follows::

$$V = \mathbf{e}^T\mathbf{P}\mathbf{e} + \sum_{i=1}^{n}\sum_{j=1}^{m} \Delta\tilde{b}_{ij}^2 + \sum_{i=1}^{n}\sum_{j=1}^{L} \tilde{w}_{ij}^2 + \sum_{i=1}^{n} \tilde{d}_i^2. \tag{14}$$

The Eq. (4) will be stable if the derivative (14) $\dot{V} < 0$. From (14), we have:

$$\dot{V} = \mathbf{e}\dot{\mathbf{P}}\mathbf{e} + \mathbf{e}^T P\dot{\mathbf{e}} + 2\sum_{i=1}^{n}\sum_{j=1}^{m} \Delta\dot{\tilde{b}}_{ij}\Delta\tilde{b}_{ij} + 2\sum_{i=1}^{n}\sum_{j=1}^{L} \dot{\tilde{w}}_{ij}\tilde{w}_{ij} + 2\sum_{i=1}^{n} \dot{\tilde{d}}_i\tilde{d}_i. \tag{15}$$

Substitute (4) into (15):

$$\begin{aligned}\dot{V} = {} & \mathbf{e}^T\left(\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A}\right)\mathbf{e} + 2\mathbf{u}^T\boldsymbol{\Delta}\tilde{\mathbf{B}}^T\mathbf{P}\mathbf{e} + 2\mathbf{e}^T\mathbf{P}\tilde{\mathbf{f}}\,(\mathbf{x}) + 2\mathbf{e}^T\mathbf{P}\tilde{\mathbf{d}}\,(t) + \\ & + 2\sum_{i=1}^{n}\sum_{j=1}^{m} \Delta\dot{\tilde{b}}_{ij}\Delta\tilde{b}_{ij} + 2\sum_{i=1}^{n}\sum_{j=1}^{L} \dot{\tilde{w}}_{ij}\tilde{w}_{ij} + 2\sum_{i=1}^{n} \dot{\tilde{d}}_i\tilde{d}_i.\end{aligned} \tag{16}$$

From (16) and (9), we have:

$$\begin{aligned}\dot{V} = {} & \mathbf{e}^T\left(\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A}\right)\mathbf{e} + 2\mathbf{e}^T\mathbf{P}\boldsymbol{\varepsilon} + 2(\mathbf{u}^T\boldsymbol{\Delta}\tilde{\mathbf{B}}^T\mathbf{P}\mathbf{e} + \sum_{i=1}^{n}\sum_{j=1}^{m} \Delta\dot{\tilde{b}}_{ij}\Delta\tilde{b}_{ij}) + \\ & + 2(\mathbf{e}^T\mathbf{P}\begin{bmatrix} \sum_{j=1}^{L} \tilde{w}_{1j}\phi_{ij}\,(\mathbf{x}) \\ \vdots \\ \sum_{j=1}^{L} \tilde{w}_{nj}\phi_{ij}\,(\mathbf{x}) \end{bmatrix} + \sum_{i=1}^{n}\sum_{j=1}^{L} \dot{\tilde{w}}_{ij}\tilde{w}_{ij}) + 2(\mathbf{e}^T\mathbf{P}\tilde{\mathbf{d}}\,(t) \ + 2\sum_{i=1}^{n} \dot{\tilde{d}}_i\tilde{d}_i).\end{aligned} \tag{17}$$

Substitute (11), (12), and (13) into (17)):

$$\dot{V} = \mathbf{e}^T\left(\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A}\right)\mathbf{e} + 2\mathbf{e}^T\mathbf{P}\varepsilon. \tag{18}$$

The Eq. (18) can be written:

$$\dot{V} = -\mathbf{e}^T\mathbf{Q}\mathbf{e} + 2\sum_{i=1}^{n} \varepsilon_i\bar{\mathbf{P}}_i\mathbf{e}, \tag{19}$$

$\mathbf{Q} = -(\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A})$, $\bar{\mathbf{P}}_i$ is the i-th row of the matrix **P**.

Using inequality transformations [12], the Eq. (19) can be written:

$$\dot{V} = -\mathbf{e}^T \mathbf{Q}\mathbf{e} + 2\sum_{i=1}^{n} \varepsilon_i \bar{\mathbf{P}}_i \mathbf{e} < -r_{\min}(Q)\|\mathbf{e}\|^2 + 2\sum_{i=1}^{n} \varepsilon_i \left\|\bar{\mathbf{P}}_i\right\| \|\mathbf{e}\|. \tag{20}$$

Substitute (10) into (20), we have $\dot{V} < 0$, the Eq. (4) is stable. □

The expressions (11), (12), and (13) of the Theorem 1 contain identification algorithms $\Delta\mathbf{B}$, $\mathbf{f}(\mathbf{x})$, and $\mathbf{d}(\mathbf{t})$.

The Eq. (11) contains slowly variable elements, i.e. $\Delta\dot{b}_{ij} \approx 0$. The matrix $\mathbf{\Delta B}$ with uncertain parameters is identified by the matrix $\mathbf{\Delta\hat{B}}$ using the update law:

$$\Delta\dot{\hat{b}}_{ij} = u_j \bar{\mathbf{P}}_i \mathbf{e}. \tag{21}$$

From (7) and (12), because of $w_{ij}^* = const$, we have $\dot{w}_{ij}^* = 0$. The vector $\hat{\mathbf{f}}(\mathbf{x})$ for identification of the nonlinear function $\mathbf{f}(\mathbf{x})$ can be written:

$$\hat{f}_i(\mathbf{x}) = \sum_{j=1}^{L} \hat{w}_{ij}\phi_{ij}(\mathbf{x}), i = \overline{1, n}. \tag{22}$$

The update weights can be defined:

$$\dot{\hat{w}}_{ij} = \bar{\mathbf{P}}_i \mathbf{e}\phi_{ij}(\mathbf{x}). \tag{23}$$

From (13), because of slow-varying external disturbance $\dot{d}(t) \approx 0$. The vector $\hat{\mathbf{d}}(t)$ for identification of $\mathbf{d}(t)$ can be written:

$$\dot{\hat{d}}_i(t) = \bar{\mathbf{P}}_i \mathbf{e}. \tag{24}$$

The received results from (21), (22), (23), and (24) are used to synthesis the compensation control law $\mathbf{u}_c$.

The Eq. (1) can be again written as follows:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{I}\mathbf{f}_{\Sigma}(t), \tag{25}$$

where $\mathbf{f}_{\Sigma}(t) = \mathbf{\Delta B}\mathbf{u} + \mathbf{f}(\mathbf{x}) + \mathbf{d}(t), \mathbf{f}_{\Sigma}(t) = [f_1^{\Sigma}, f_2^{\Sigma}, ..., f_n^{\Sigma}]^T; \mathbf{I}^{n\times n}$ has main diagonal elements $I_{ij} = 1,\ i = j = \overline{1, n}$ are rows which corresponds to the vector $\mathbf{f}_{\Sigma}(t)$ in the case $|f_i^{\Sigma}| \neq 0$; other elements $I_{ij} = 0$ in the case $i \neq j$ and $|f_i^{\Sigma}| = 0$.

Substitute (2) into (25):

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}_{smc} + \mathbf{B}\mathbf{u}_c + \mathbf{I}\mathbf{f}_{\Sigma}(t). \tag{26}$$

The vector $\mathbf{u}_c$ can be selected:

$$\mathbf{u}_c = -\mathbf{H}\hat{\mathbf{f}}_{\Sigma}(t), \tag{27}$$

$$\hat{\mathbf{f}}_{\Sigma}(t) = \Delta\hat{\mathbf{B}}\mathbf{u} + \hat{\mathbf{f}}(\mathbf{x}) + \hat{\mathbf{d}}(t);\tag{28}$$

$\Delta\hat{\mathbf{B}}$, $\hat{\mathbf{f}}(\mathbf{x})$, and $\hat{\mathbf{d}}(t)$ are presented in (21), (22), (23), and (24).

Substitute (27) into (26):

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}_{smc} - \mathbf{B}\mathbf{H}\hat{\mathbf{f}}_{\Sigma}(t) + \mathbf{I}\mathbf{f}_{\Sigma}(t).\tag{29}$$

From (29) we can see that uncertain elements will be compensated with the condition:

$$-\mathbf{B}\mathbf{H}\hat{\mathbf{f}}_{\Sigma}(t) + \mathbf{I}\mathbf{f}_{\Sigma}(t) = 0.\tag{30}$$

The Eq. (30) will be satisfied with the following condition:

$$\mathbf{B}\mathbf{H} = \mathbf{I}.\tag{31}$$

The Eq. (31) will be satisfied with:

$$\mathbf{H} = \mathbf{B}^{+}.\tag{32}$$

where $\mathbf{B}^{+}$ is the pseudo-inverse matrix of $\mathbf{B}$.

Thus, the article has synthesized the compensation control law $\mathbf{u_c}$ (27) with identification vectors $\hat{\mathbf{f}}_{\Sigma}(t)$ (28), $\mathbf{H}$ (32).

Using the compensation control law (27), the Eq. (29) can be written:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}_{smc}.\tag{33}$$

Thus, in this section, the identification and compensation control law $\mathbf{u_c}$ (27) for the uncertain components of (1) have been presented, and then (1) is rewritten to (33). For (33), the control law is synthesized based on the sliding mode control.

4 Synthesis of the Sliding Mode Control Law

The error vector between the state vector $\mathbf{x}$ and the desired state vector $\mathbf{x}_d$:

$$\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_d \rightarrow \mathbf{x} = \tilde{\mathbf{x}} - \mathbf{x}_d.\tag{34}$$

Substitute (34) into (33), we have:

$$\dot{\tilde{\mathbf{x}}} = \mathbf{A}\tilde{\mathbf{x}} + \mathbf{B}\mathbf{u}_{smc} + \mathbf{A}\mathbf{x}_d - \dot{\mathbf{x}}_d.\tag{35}$$

For (35), the hyper sliding surface is chosen as follows [13]:

$$\mathbf{s} = \mathbf{C}\tilde{\mathbf{x}},\tag{36}$$

where $\mathbf{C}$ is the parameter matrix of hyper sliding surface, $\mathbf{s} = [s_1, s_2, ..., s_n]^T$.

The next problem is to define the control law $\mathbf{u}_{smc}$ which ensures movement of the system (35) towards the hyper sliding surface (36) and keep it there.

The control signal $\mathbf{u}_{smc}$ can be written by:

$$\mathbf{u}_{smc} = \begin{cases} \mathbf{u}_s & \text{if} \quad \mathbf{s} \neq 0 \\ \mathbf{u}_{eq} & \text{if} \quad \mathbf{s} = 0 \end{cases}, \tag{37}$$

$\mathbf{u}_s$ is the control signal that moves the system (35) towards the hyper sliding surface (36); $\mathbf{u}_{eq}$ is the equivalent control signal that keeps the system (35) on the hyper sliding surface (36).

The Eq. (37) can be rewritten as:

$$\mathbf{u}_{smc} = \mathbf{u}_{eq} + \mathbf{u}_s, \tag{38}$$

$\mathbf{u}_{eq}$ is defined in [13]:

$$\dot{\mathbf{s}} = \mathbf{C}\dot{\tilde{\mathbf{x}}} = 0. \tag{39}$$

From (35) and (39), we have:

$$\mathbf{C}\left(\mathbf{A}\tilde{\mathbf{x}} + \mathbf{B}\mathbf{u}_{eq} + \mathbf{A}\mathbf{x}_d - \dot{\mathbf{x}}_d\right) = 0. \tag{40}$$

From (40), the equivalent control signal can be defined as follows:

$$\mathbf{u}_{eq} = -[\mathbf{CB}]^{-1}\left[\mathbf{CA}\tilde{x} + \mathbf{CA}\mathbf{x}_d - \mathbf{C}\dot{\mathbf{x}}_d\right]. \tag{41}$$

Next, we define the control signal $\mathbf{u}_s$ that moves the system (35) towards the hyper sliding surface (36).

For the hyper sliding surface (36), the Lyapunov function can be selected by:

$$V = \frac{1}{2}\mathbf{s}^T\mathbf{s}. \tag{42}$$

Condition for the existence of slip mode can be written:

$$\dot{V} = \mathbf{s}^T\dot{\mathbf{s}} < 0. \tag{43}$$

Substitute (35) and (38) into (43), with attention to (39), (40) we have:

$$\dot{V} = \mathbf{s}^T\left[\mathbf{C}\left(\mathbf{A}\tilde{\mathbf{x}} + \mathbf{B}\mathbf{u}_{eq} + \mathbf{A}\mathbf{x}_d - \dot{\mathbf{x}}_d\right) + \mathbf{CB}\mathbf{u}_s\right] < 0. \tag{44}$$

Inequality (43) can be written as:

$$\mathbf{s}^T\left[\mathbf{CB}\mathbf{u}_s\right] < 0. \tag{45}$$

So to satisfy the condition (43), the control signal from (45) can be defined as follows:

$$\mathbf{u}_s = -[\mathbf{CB}]^{-1}\left[\delta \mathrm{sgn}\left(s_1\right), \delta \mathrm{sgn}\left(s_2\right), ..., \delta \mathrm{sgn}\left(s_n\right)\right]^T, \tag{46}$$

δ is a positive coefficient. Substituting (41) and (46) into (37), the control signal can be defined by $\mathbf{u}_{smc}$ as follows:

$$\mathbf{u}_{smc} = \begin{cases} -[\mathbf{CB}]^{-1}\left[\delta \mathrm{sgn}\left(s_1\right), \delta \mathrm{sgn}\left(s_2\right), ..., \delta \mathrm{sgn}\left(s_n\right)\right]^T & if \quad \mathbf{s} \neq 0 \\ -[\mathbf{CB}]^{-1}\left[\mathbf{CA}\tilde{\mathbf{x}} + \mathbf{CA}\mathbf{x}_d - \mathbf{C}\dot{\mathbf{x}}_d\right] & if \quad \mathbf{s} = 0 \end{cases}. \tag{47}$$

Finally, the control signals (27) and (47) are used for (2), and the control laws of (1) have been synthesized successfully.

5 Results and Discussion

Simulations are implemented on the Matlab environment for the controller (2) where parameter matrix, nonlinear function vectors, disturbance vectors of the system (1) are defined as follows:

$$\begin{gathered}\mathbf{A} = \begin{bmatrix} -3.7376 & 0.0779 \\ 2.3515 & -4.1702 \end{bmatrix}; \mathbf{B} = \begin{bmatrix} 0.7014 & 0.5629 \\ 0.7248 & 0.4541 \end{bmatrix}; \mathbf{\Delta B} = \begin{bmatrix} 0.2104 & 0.1689 \\ 0.2174 & 0.1362 \end{bmatrix}; \\ \mathbf{f}\left(\mathbf{x}\right) = \begin{bmatrix} 0.02 \sin\left(x_1\right) \sin\left(x_2\right) \\ 0.02 x_1^2 \end{bmatrix}; \mathbf{d}\left(t\right) = \begin{bmatrix} 0.2 \sin\left(0.5t\right) \\ 0.2 \cos\left(0.7t + 2\right) \end{bmatrix}. \end{gathered} \tag{48}$$

With the desired signal $\mathbf{x_d} = [1.5, 1.0]^T$. The simulation results are shown in Figs. 2, 3 and 4.

The results of the identification of variable parameter components, nonlinear function vectors, and external disturbance are shown in Fig. 2. The results after using the compensation signal from the identification rule for uncertain components are presented in Fig. 3. From Figs. 2 and 3, we can see that the uncertain components are identified and compensated with an asymptotic error of zero. Figure 4 depicts responses of the system which present the result of tracking the state vectors of the system with the desired signal vector. These simulation results once again prove the correctness and effectiveness of the proposed control law.

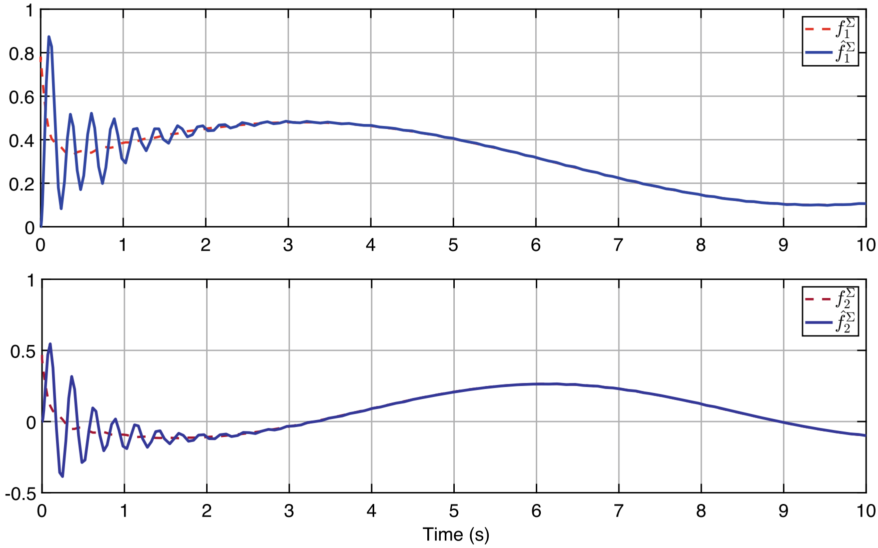

Fig. 2 The identification vectors $\hat{\mathbf{f}}_{\Sigma}$ (28)

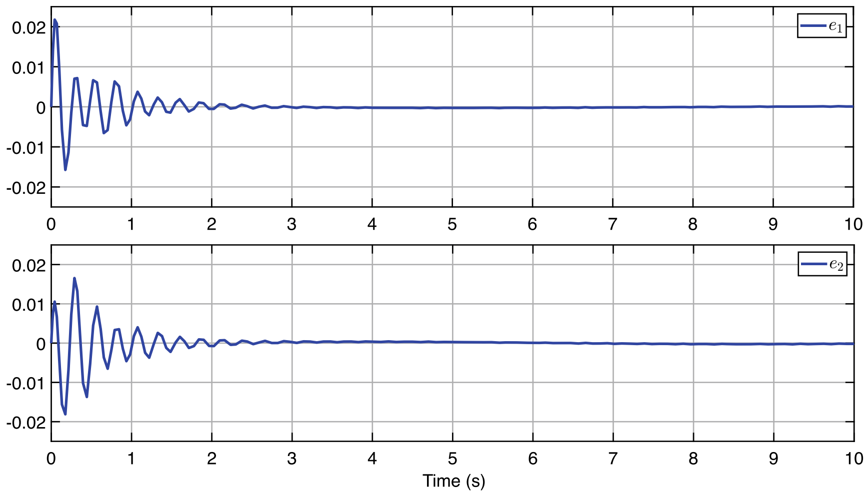

Fig. 3 The error between (1) and linear model (33) with compensation for uncertain components

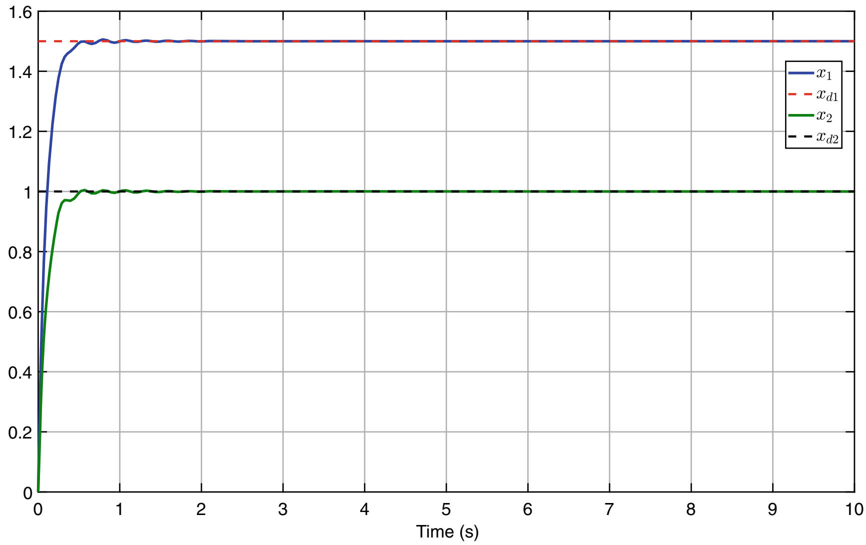

Fig. 4 Responses of the system for the desired signals $\mathbf{x}_d$

6 Conclusion

The article has synthesized the controller for a class of nonlinear objects. Lyapunov stability theory is used to design the adaptive update law which allows identifying uncertain parameter components, nonlinear function vectors, and external disturbance. The identification results are used to generate adaptive control rules that compensate for the uncertain components. And then, the linear part of the control law is synthesized based on sliding mode control. The simulation results show that the variable parameter components, nonlinear function vectors, and external disturbance are identified and compensated according to the algorithm proposed by the article; the output vector of the system tracks to the desired set signal vector with high controllability.

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