2-D Fir Filter Design And Its Applications In Removing Impulse Noise In Digital Image

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Abstract: This paper conducts research into the design of 2-D FIR filter using the REMEZ algorithm and the applications of 2-D FIR filter in the digital image processing field. In the procedure, the author has studied the theories to create algorithm and simulation utilizing MATLAB software to design 2-D FIR filter and therefore, tested its applications in restoring digital image having impulse noise.

Keywords: DSP, 2-D FIR filter, REMEZ, image processing.

I. INTRODUCTION

In most areas, the filter is perceived as the heart of the system. In another word, there is not a system that does not use the filter. The use of filters designed on the basis of signal processing has been of very early interest. Previously, derived from signal analysis and signal processing in a time domain, the authors studied the design of the 1-D digital filter. Since then, it has been developed into the 2-D digital filter for signal processing in the spatial domain.

There are two types of digital filters: finite impulse response (FIR) and infinite impulse response (IIR). Most of the authors are interested in the design of the FIR filter as the FIR filter is a non-responsive, finite and stable filter. Hence, the design is simpler. The FIR filter design can be achieved by methods such as FFT, window method and optimal approximation method of the REMEZ algorithm.

For the 1-D FIR filter design, a number of authors are attentive in the REMEZ algorithm based on the Chebyshev’s optimal approximation theory. Nevertheless, when designing a 2-D FIR filter or developing a multi-dimensional FIR filter (M-D), the use of the REMEZ algorithm meets a number of hindrances due to the set of cos functions used in 2D approximation (M-D) that does not satisfy the conditions in the domain of interest. Furthermore, the technique of utilizing modified algorithms for the expansion of 2-D (MD) of Chebyshev’s approximation calculations is quite complex.

In the image processing field, a large number of studies have applied filter types to resolve problems including impulse noise. For the impulse noise removal, they use nonlinear filters such as the Median filter. However, this paper chooses the 2-D FIR filter using REMEZ algorithm, which is a linear filter, to handle the impulse noise in digital images. This is a quite bold idea to be considered as a pilot study. Thus, this topic has a real scientific significance.

II. THE 2-D FIR FILTER

A. Overview of the 2-D FIR filter

The 2-D FIR filter is a linear, discrete, and invariant system in the spatial domain (m, n). The 2D differential equation that defines the invariant linear filter has the following form:

$$\sum_{m_1=0}^{N_1-1}\sum_{n_1=0}^{N_2-1} a_{m_1,n_1}(m,n)y(m-m_1,n-n_1) = \sum_{m_2=0}^{M_1-1}\sum_{n_2=0}^{M_2-1} b_{m_2,n_2}(m,n)x(m-m_2,n-n_2) \tag{2.1}$$

where $\{x(m,n)\}$ is the input signal, $\{y(m,n)\}$ is the output signal, and $\{a(m,n)\}$, $\{b(m,n)\}$ are coefficients that define the filter's transfer function.

In the plane (z_1,z_2), the filter (2.1) is described by the transfer function in the form of a rational fraction:

$$H(z_1,z_2) = \frac{\sum_{m=0}^{M_1-1}\sum_{n=0}^{M_2-1} b(m,n)z_1^{-m}z_2^{-n}}{\sum_{m=0}^{N_1-1}\sum_{n=0}^{N_2-1} a(m,n)z_1^{-m}z_2^{-n}} \tag{2.2}$$

If a(0,0)=1 and a(m,n)=0 for m, n # 0 (2.2) is written as:

$$H(z_1,z_2) = \sum_{m=0}^{M_1-1}\sum_{n=0}^{M_2-1} b(m,n)z_1^{-m}z_2^{-n} \tag{2.3}$$

In that case, the difference equation has the form:

$$y(m,n) = \sum_{m_2=0}^{M_1-1}\sum_{n_2=0}^{M_2-1} b_{m_2,n_2}(m,n)x(m-m_2,n-n_2) \tag{2.4}$$

(2.3) and (2.4) are the transfer function and the difference equation of the 2-D FIR filter, respectively.

B. Designing the 2-D FIR filter based on the REMEZ algorithm.

The 2-D FIR filter is designed according to the REMEZ algorithm and is based on the optimal approximation of Chebyshev.

The Chebyshev optimal 2-D filter design means that the filter criteria allow for changes to minimize the max errors in the approximate areas.

The actual FIR filter characterized by the technical parameters in the constant frequency domain ω has four main parameters:

- δ_p : passband ripple
- δ_s : stopband ripple
- ω_p : limited frequency (amplitude frequency) of passband
- ω_s : limited frequency (amplitude frequency) of stopband

For one actual FIR filter, the ripples do not exceed δ_p in passband and δ_s in stopband.

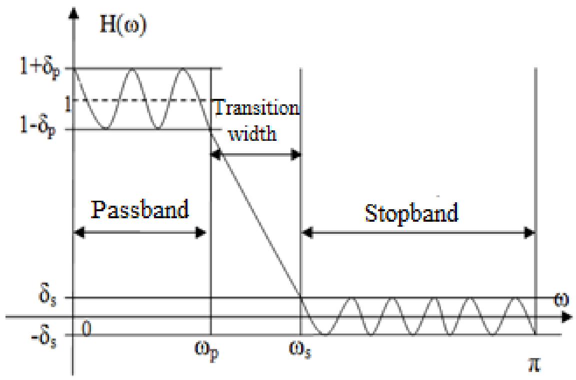

Fig.1 Filter criteria

In fact, when constructing the filter, attenuations in the passband (A_p) and the stopband (A_s) are taken into account to calculates the ripples δ_p, δ_s according to the formula (2.5):

$$\delta_p = \frac{10^{A_p/20} - 1}{10^{A_p/20} + 1}$$
$$\delta_s = 10^{-A_s/20} \quad (2.5)$$

The optimal design indicates that the energy of the lateral wave is minimal, the ripples in the passband, the transition band gap, and the stopband are small; in other words, it is as ideal as possible. The optimasl FIR filter design leads to the Chebyshev approximation problem. Since there exists the sinusoidal oscillation around the ideal function in Chebyshev norm, it is possible to minimize the maximum absolute values which can be called as the optimal approximation. The optimization here means the approximation of the filter criteria. With the optimization program on the computer, the maximum approximate error will be minimal. Parks and Mc.Clellan offered the optimal method based on REMEZ algorithm. This is known to be the most widely used and efficient FIR filter design tool. The REMEZ exchange algorithm is based on the Chebyshev approximation problem; Independent points are spread on the frequency axis in an appropriate way so that the maximum approximate error reaches its smallest. The smaller the ripple can be, the closer it can get to the ideal state.

General principles of the method are illustrated as follows:

+ Choose the ideal filter, which means to select the frequency response of the filter $A\left(e^{j\omega}\right)$, then select the weighting function $\mathrm{W}(e^{j\omega})$ (based on specifications of the actual filter), finally, pick out the level of the N filter.

+ Determine the approximate problem: find the approximation of the weighting function $\widehat{\mathrm{W}}\left(e^{j\omega}\right)$, the frequency response of the strain filter $\widehat{A}\left(e^{j\omega}\right)$ và $P\left(e^{j\omega}\right)$ is the polynomial of the $\cos\omega n$ function.

+ Solve the approximation problem utilizing the REMEZ exchange algorithm.

+ Calculate the coefficients of the filter.

The algorithm procedure is specifically demonstrated as follows:

- **Step 1**: Assume that a low pass filter is designed with $\delta_s > \delta_p$.

+ Choose $$A\left(e^{j\omega}\right) = \begin{cases} 1 & \textit{in stopband} \\ 0 & \textit{in passband} \end{cases} \quad (2.6)$$

+ Choose $$W\left(e^{j\omega}\right) = \begin{cases} \dfrac{\delta_s}{\delta_p} = \dfrac{\delta}{\delta_p} & \textit{in stopband} \\ 1 & \textit{in passband} \end{cases} \quad (2.7)$$

where $\delta = \max\left[\delta_s, \delta_p\right]$ (since this filter has $\delta_s > \delta_p => \delta = \delta_s$)

+ Choose N (N is on odd number with FIR filter type 1): the Kaiser approximation formula is employed to find the filter level:

$$N \cong \frac{-20\log\sqrt{\delta_p \delta_s} - 13}{14.6(\omega_s - \omega_p)/2\pi} \quad (2.8)$$

- **Step 2**: Determine the approximate problem.

$$\mathrm{E}\left(e^{j\omega}\right) = \widehat{\mathrm{W}}\left(e^{j\omega}\right).\left[\widehat{A}\left(e^{j\omega}\right) - P\left(e^{j\omega}\right)\right] \quad (2.9)$$

Consider the FIR filter type 1: with a known fixed function $Q\left(e^{j\omega}\right)=1$. Then:

$$\widehat{W}\left(e^{j\omega}\right)=W\left(e^{j\omega}\right).Q\left(e^{j\omega}\right)=W\left(e^{j\omega}\right) \tag{2.10}$$

$$\widehat{A}\left(e^{j\omega}\right)=\frac{A\left(e^{j\omega}\right)}{Q\left(e^{j\omega}\right)}=A\left(e^{j\omega}\right) \tag{2.11}$$

Polynomial:

$$P\left(e^{j\omega}\right)=\sum_{n=0}^{\frac{N-1}{2}}\alpha'(n)\cos\omega n \tag{2.12}$$

where: $\alpha'(n)=\alpha(n)$ is a set of coefficients and put $R=\frac{N-1}{2}$

$$\Rightarrow P(e^{j\omega})=\sum_{n=0}^{R}\alpha(n)\cos\omega n \tag{2.13}$$

- **Step 3**: Use the REMEZ exchange algorithm to solve the approximate problem.
According to the alternation theorem, in the frequency domain, it is requisite to select discrete frequency point R + 2. In the R + 2 set, if the initial selection frequency is $\{\omega_i\}$, the error $E\left(e^{j\omega}\right)$ must reach the extreme value $\pm\delta$ at these frequencies and the sign of δ has to be interleaved (alternating). That means the following equation has to be solve:

$$E\left(e^{j\omega_i}\right)=\widehat{W}\left(e^{j\omega_i}\right).\left[\widehat{A}\left(e^{j\omega_i}\right)-P\left(e^{j\omega_i}\right)\right]=(-1)^i\delta \tag{2.14}$$

and $P(e^{j\omega_i})=\sum_{n=0}^{R}\alpha(n)\cos\omega_i n$ (2.15)

$$\widehat{A}\left(e^{j\omega_i}\right)-P\left(e^{j\omega_i}\right)=\frac{(-1)^i}{\widehat{W}\left(e^{j\omega_i}\right)}\delta \tag{2.16}$$

$$P\left(e^{j\omega_i}\right)+\frac{(-1)^i}{\widehat{W}\left(e^{j\omega_i}\right)}\delta=\widehat{A}\left(e^{j\omega_i}\right) \tag{2.17}$$

The equation (2.17) can be written in matrix form as follows:

$$\begin{bmatrix} 1 & \cos\omega_0 & \cos 2\omega_0 & \dots & \cos R\omega_0 & 1/\widehat{W}\left(e^{j\omega_0}\right) \\ 1 & \cos\omega_1 & \cos 2\omega_1 & \dots & \cos R\omega_1 & -1/\widehat{W}\left(e^{j\omega_0}\right) \\ . & & & & & \\ . & & & & & \\ . & & & & & \\ 1 & \cos\omega_R & \cos 2\omega_R & \dots & \cos R\omega_R & (-1)^R/\widehat{W}\left(e^{j\omega_R}\right) \\ 1 & \cos\omega_{R+1} & \cos 2\omega_{R+1} & \dots & \cos R\omega_{R+1} & (-1)^{R+1}/\widehat{W}\left(e^{j\omega_{R+1}}\right) \end{bmatrix} \begin{bmatrix} \alpha(0) \\ \alpha(1) \\ . \\ . \\ . \\ \alpha(R) \\ \delta \end{bmatrix} = \begin{bmatrix} \widehat{A}\left(e^{j\omega_0}\right) \\ \widehat{A}\left(e^{j\omega_1}\right) \\ . \\ . \\ . \\ \widehat{A}\left(e^{j\omega_R}\right) \\ \widehat{A}\left(e^{j\omega_{R+1}}\right) \end{bmatrix} \tag{2.18}$$

From the set of extreme frequencies $\{\omega_i\}$, $P\left(e^{i\omega}\right)$ and δ are ascertained, then, the error $E\left(e^{i\omega}\right)$ is calculated. If $\left|E\left(e^{j\omega}\right)\right|\le\delta$ for $\forall\omega$, it indicates that the optimal solution is found. If $\left|E\left(e^{j\omega}\right)\right|>\delta$ for some radom frequency, a new set of extreme frequencies R+2 has to be selected, in which $E\left(e^{j\omega}\right)$ reaches the extreme, therefore, new value of δ is calculated according to this new set of frequencies,

$P\left(e^{j\omega}\right)$ also has to be recalculated and $\left|E\left(e^{j\omega}\right)\right|$ need to be reassessed. This process is repeated until $\left|E\left(e^{j\omega}\right)\right| \le \delta$ for all frequencies in the selected frequency set. At this stage, the result is considered the optimal solution.

- **Step 4**: Determine the impulse response $h_d(n)$ of the actual filter.

The identification of $h_d(n)$ can be implemented in two ways:

+ Knowing $P\left(e^{j\omega}\right)$ (according to the optimal solution), $h_d(n)$ is directly computed without considering the intermediate step $\alpha(n)$. For FIR filter type 1, the following formula is applied:

$$h_d(n) = \begin{cases} \dfrac{A(0)}{N} + \dfrac{2}{N}\sum_{k=1}^{\frac{N-1}{2}} (-1)^k A(k)\cos\left[\dfrac{\pi}{N}k(2n+1)\right] & \textit{with } 0 \le n \le N-1 \\ 0 & \textit{with the remaining } n \end{cases} \tag{2.19}$$

+ Knowing $P\left(e^{j\omega}\right)$ (based on the optimal solution), the sample $P\left(e^{j\omega}\right)$ is taken by M point, subsequently, coefficients $\alpha(n)$ are determined by IDFT, from which $h_d(n)$ of the filter is established.

In 1972, Park and McClellan wrote a program to design a 1-D linear phase FIR filter based on the Chebyshev approximation standard implemented by the REMEZ algorithm. The design of the 2-D FIR digital filter by REMEZ algorithm also performs the same steps as for the 1-D FIR filter. In this design procedure, the 1-D FIR filters that have the frequency response $H(\omega)$ are converted into the 2-D FIR digital filters with the 2-D frequency response $H(\omega_1, \omega_2)$ by the following transformations:

$$H(\omega_1, \omega_2) = H(\omega)\Big|_{\cos\omega = T(\omega_1, \omega_2)} \tag{2.20}$$

where $H(\omega)$ is the frequency response of the 1-D FIR filter that has the length N and the frequency response $h(n)$. $T(\omega_1, \omega_2)$ is the discrete-space fourier transform of the transformed matrix $t(m,n)$, which is calculated as:

$$T(\omega_1, \omega_2) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} t(m,n)e^{-j(\omega_1 m + \omega_2 n)} \tag{2.21}$$

Hence, the impulse response $h(m,n)$ of the 2-D filter, which needs to be constructed, is the inverse Fourier transform of the frequency response $H(\omega_1, \omega_2)$:

$$h(m,n) = \frac{1}{4\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} H(\omega_1, \omega_2)e^{j(\omega_1 m + \omega_2 n)}d\omega_1 d\omega_2 \tag{2.22}$$

In MATLAB, the ***ftrans2*** function is utilized to perform the above transformations. The 1-D FIR filter design by REMEZ algorithm uses the ***firl*** or ***REMEZ*** function and employs the syntax $h_2 = ftrans2(h_1, T)$ to convert into the 2-D FIR digital filter (h_1 is the 1-D FIR filter and h_2 is the 2-D FIR filter, which are obtained by the transformation T).

The design of the FIR filter according to REMEZ algorithm is executed based on the algorithmic flowchart shown in Figure 2 [1] below:

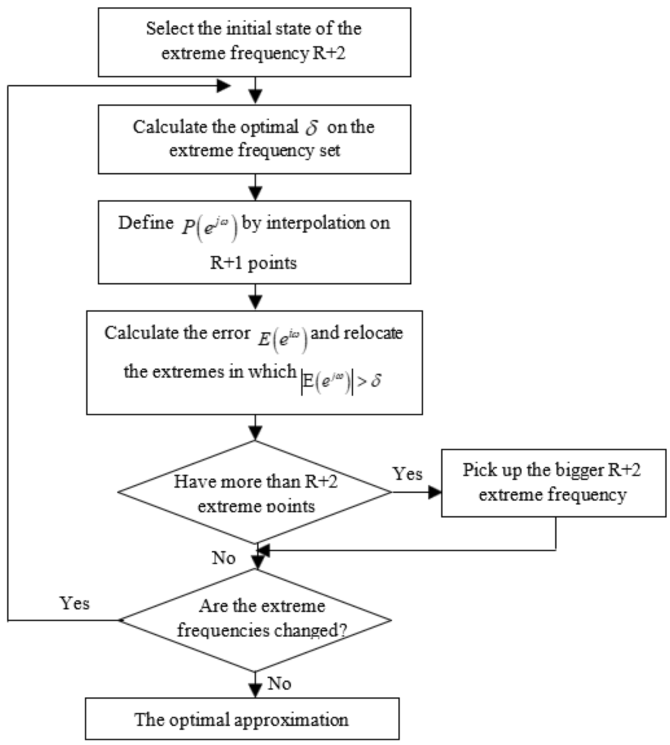

Fig.2 The algorithmic flowchart

C. Applying the 2-D FIR filter in processing digital images that have impulse noise

Impulse noise is a quite distinctive kind of noise that can be produced for various reasons. It often appears as discrete bright spots and dark spots on the image. Digital images represented by a two-dimensional matrix of real numbers or complex numbers include a finite number of bits. A continuous two-dimensional signal in space is referred to as a continuous image in the real number field and is denoted as f (x, y) (the value of f (x, y) that continuously appears in the range of (-∞ , + ∞)). Since the image signal has the characteristic of the two-dimensional space, it is necessary to design a 2-D FIR digital filter to process the image. The procedure of removing impulse noise in a digital image using the 2-D FIR digital filter is illustrated in Figure 3.

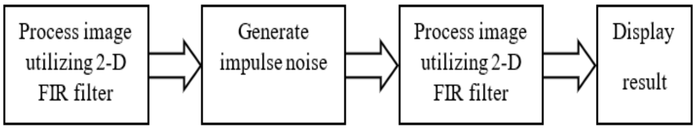

Fig.3 Simulation of OFDM system using MATLAB SIMULINK software

The process begins by importing the original image. Then the impulse noise is generated for the original image. The image is passed through the designed 2-D FIR filter. After the impulse noise image is applied through the filter, the final image results to be sharper, minimizing the noise impact on the image.

III. RESULTS AND DISCUSSION

From the theoretical study on the design of FIR filters according to REMEZ algorithm, the author has introduced an algorithmic flowchart and built a simulation program on the MATLAB software. The author proposes a programmable interface for designing several types of filters with different design criteria. Within the paper framework, the author gives only some illustrative results.

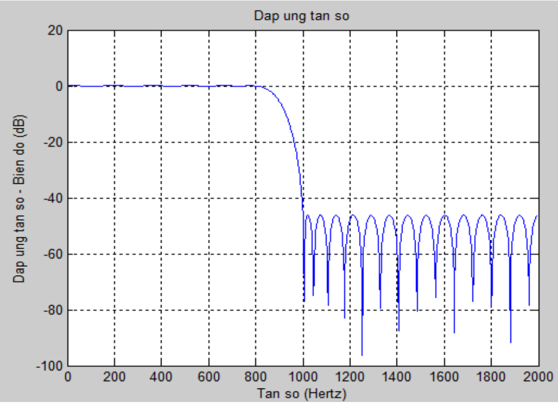

Fig.4 Frequency response – amplitude of the 1-D FIR filter

Figure 4 shows results of the 1-D low-pass filter. In the passband [0,800Hz], the ripples are very small and almost straight, which denote that it reaches the ideal filter. Utilizing a switch from the 1-D FIR filter to the 2-D FIR digital filter offers a more general and comprehensive outlook at the two-dimensional low-pass FIR filter as shown in Figure 2.

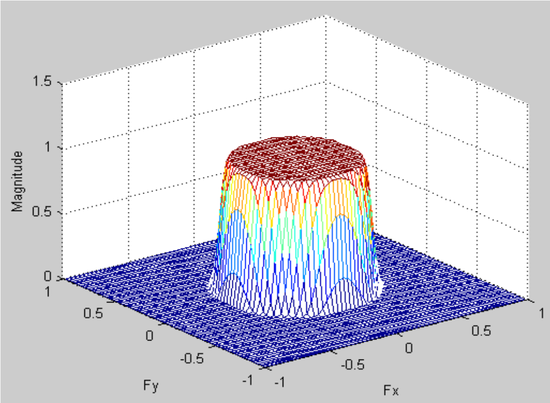

Fig.5 Frequency response - amplitude of the 2-D FIR filter

In order to test the image processing using the designed 2-D FIR filter, the impulse noise function is affected on the original image and therefore, produces white spots and black spots (Figure 6). A white spot on the image that looks like a "salt" as the pixel has the maximum value (the light intensity is 255). In contrast, a black spot on the image looks like a "pepper" since the pixel has a minimum value (the light intensity is zero).

Fig.6 An image impacted by the impulse noise

Processed through the designed 2-D FIR digital filter, the image is smoother, white spots and black spots on the image caused by the impulse noise are minimized (Figure 7). As a result, the 2-D FIR filter designed by REMEZ algorithm has processed quite well the impulse noise image.

Fig.7 The image after filter processing

IV. CONCLUSION

The paper presents simulation results on MATLAB software designed from the 1-D FIR filter to the 2-D FIR filter according to REMEZ algorithm and the 2-D FIR digital filter application on the impulse noise removal. The most outstanding feature of this design approach is to ground on the Chebyshev approximation. On that account, the maximum error of the filter is minimal. The 2-D FIR digital filter can be applied to various problems in the field of image processing as well as in a number of other areas. Nevertheless, the simulation in this paper is limited to the processing of the impulse noise. In the future, the author will carry out further study on the simulation of 2-D FIR filter application on noise image processing with various types of noise and the sharpening in image processing.

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