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Editors

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Finding "Optimal" Stable Marriages with Ties via Local Search

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***Abstract*—The stable marriage problem with ties (SMT) is a variant of the stable marriage problem in which people are permitted to express ties in their preference lists. The problem have been received many attentions of researchers due to useful applications in practice. Many methods proposed for solving the problem are to only find a stable matching of a given instance. In this paper, we present a local search based algorithm which attempts to find *egalitarian* and *sex-equal* stable matchings. The algorithm is based on the dominance of stable matchings with respect to men. Particularly, starting at the man-optimal stable matching the algorithm processes through a sequence of stable matchings to seek the *egalitarian* and *sex-equal* stable matchings. For each a stable matching of the sequence, a neighbor set of is constructed using a breaking marriage operation. The best neighbor stable matching by means of a cost function is selected to continue the search. Experimental results show that our algorithm is significant.**

I. Introduction

The stable marriage problem was first introduced by Gale and Shapley [3] and has a wide range of practical applications such as the Evolution of the Labor Market for Medical Interns and Residents [18], the Student-Project Allocation problem [1] and the Stable Roommates problem [2].

The Classical Stable Marriage Problem

An instance I of the classical stable marriage problem (SMP) of size n involves n men and n women. Each man ranks n women to give himself a preference list, and similarly each woman ranks n man to also give herself a preference list. Problem is then to find an one-one correspondence matching M in I such that the matching is *stable*, meaning that there are no two people of opposite sex who would both matched each other than their current partners. Such a matching, fortunately, always exists and can be found in polynomial time. We denote a matching $M = \{(m_1, w_1), (m_2, w_2), \ldots, (m_n, w_n)\}$ which is a set of pairs of man and woman. Let $(m, w) \in M$, then we say that m and w are partners in M and denote by $m = M(w)$ and $w = M(m)$.

Definition 1 (see [6], [14])**.** A man m and a woman w form a blocking pair in M if m and w are not partners in M, but m prefers w to $M(m)$ and w prefers m to $M(w)$.

A matching M is stable if there is no blocking pair in M, otherwise M is unstable. Let us denote by $mr(m, w)$ and $wr(w, m)$ the rank of w in m's preference list and the rank of m in w's preference list, respectively. For a stable matching M, we define the *man cost*, denoted by $sm(M)$, and the *woman cost*, denoted by $sw(M)$, as follows:

$$sm(M) = \sum_{(m,w)\in M} mr(m, w), \tag{1}$$

$$sw(M) = \sum_{(m,w)\in M} wr(w, m). \tag{2}$$

Definition 2 (see [17])**.** Let $\mathcal{M}$ be the set of all stable matchings of an instance I of the SMP. For $M \in \mathcal{M}$, M is called to be, respectively,

- *man-optimal* if $sm(M)$
- *woman-optimal* if $sw(M)$
- *egalitarian* if $sm(M) + sw(M)$
- *sex-equal* if $|sm(M) - sw(M)|$

is minimum among all stable matchings in $\mathcal{M}$.

In order to find the man and woman-optimal matching of an instance I, Gale and Shapley gave an algorithm in $O(n^2)$ time [3]. The algorithm processes on a sequence of proposals from men to women to find the man-optimal matching. If the roles of men and women in the sequence of proposals are exchanged, the woman-optimal matching would be found. It, however, is not fair that a matching given by Gale and Shapley algorithm is one-sided optimal. The fairness is emphasized for the egalitarian and sex-equal matchings in which we attempt to obtain the balance of preferences between men and women in a stable matching. There are several approaches proposed for finding egalitarian and sex-equal matching such as genetic algorithm [17], ant colony system [19], and approximation algorithm [13].

The Stable Marriage Problem with Ties

A variant of SMP called the stable marriage problem with ties (SMT) arises when people are permitted to express ties in their preference lists. Particularly, each person does not need to rank members of the opposite sex in *strict order*. Some of those involved might be indifference among members of the opposite sex.

Regarding to stability of matchings in SMT, three definitions actually were stated which include *super stability*,

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strong stability, and *weak stability* [8]. It was indicated that a super stable matching is strongly stable, and a strongly stable matching is weakly stable (see [11]). In practice, the weak stability has received the most attention in the literature as mentioned in [14]. We now define a blocking pair and weakly stable marriages in SMT. Let M be a matching of an instance of SMT.

Definition 3 (blocking pair in SMT)**.** A man m and a woman w form a blocking pair in M if m and w are not partners in M, but m strictly prefers w to $M(m)$ and w strictly prefers m to $M(w)$.

Definition 4 (weakly stable marriages in SMT)**.** A matching M is weakly stable if there is no blocking pair in M.

We only consider the SMT for weak stability in this work. The term of stability will then be used throughout the paper to indicate the weak stability. We also note that in SMT, solutions are stable marriages, i.e., everybody is married [4].

Due to the useful in practice, SMT and SMTI (stable marriage with ties and incomplete preference lists) have received many attentions of researchers. Irwing, in 1985, proved that the Stable Roommates Problem, a version of SMP where the bipartite matching is not required, is NP-complete when the ties are allowed [7]. He then studied SMT for the super and strong stability [8]. Iwama et al., in 1999, continued to study SMT as well as SMTI [12]. Under assumptions of ties, they gave two reductions to show that SMTI always has a completely stable matching, is NP-complete. In later years, several algorithms were proposed which emphasize the performance of solving SMT and SMTI instances. In 2002, Gent and Prosser gave an empirical study of SMTI [5]. They presented first complete algorithm for SMTI by formulating the problem as a constrain programming one. A useful approach that has been applied to deal with the problems of large size, is local search. In 2013, Gelain et al. [4] proposed a local search method to speed up the process of finding solutions. Very recently, Munera et al. have addressed STMI using local search approach, based on Adaptive Search [16]. However, the methods mentioned above only find a stable matching of a given SMTI instance.

In this paper, we investigate a new local search approach to address SMT for the egalitarian or sex-equal stable matchings. Our method processes on a sequence of stable matchings to finding the egalitarian or sex-equal one. The Gale-Shapley algorithm [3] is used to obtain a man-optimal stable matching which would be the initial solution of our local search. Based on the dominance of stable matchings, a neighbor set is generated by the breaking operation [15] to choose a better stable matching by means of a cost function. The method does iteratively until the neighbor set of the current matching is empty. The simulation results show that our method is significant.

The rest of the paper is organized as follows. Section II recalls Gale-Shapley algorithm and breaking marriage operation which are used in our method. The new algorithm is given in Section III. Section IV is devoted to implementation and discussion. Finally, Section V is conclusion.

II. Background

Gale-Shapley algorithm for SMT

Gale-Shapley algorithm [3] finds the man-optimal stable matching as well as woman-optimal stable matching when the roles of men and women are exchanged. In our method, as mentioned, the man-optimal stable matching of an instance of SMT is considered as the initial solution. We, therefore, briefly describe it here.

For an instance of SMP, the algorithm starts with assigning each person to be free. At each iteration step, the algorithm chooses a free man m and finds the most preferred woman w in m's list to whom m has not proposed. If w is free then w and m become engaged. If w is engaged to m' then she rejects the man that she least prefers to engage to the other man. The rejected man then becomes free. The algorithm terminates when all men are engaged.

By discarding the ties, an instance I of SMT becomes to an instance I' of SMP. If M is a stable matching for I', then M is so for I. It follows that a stable matching of I can be obtained by Gale-Shapley algorithm [14].

Breaking marriages

The purpose of breaking marriage operation is to obtain a stable matching from another one. The operation similarly processes to Gale-Shapley algorithm. Let M be a stable matching and m be a man which is married to woman w in M. The operation, denoted by (M, m), starts by "*breaking*" the marriage of m and w. m is now free, and w is waiting for a new proposal. She only accepts a proposal from a man that she prefers to m. Then m proposes to the woman following w in his list. A sequence of proposals, rejections, and acceptance by Gale-Shapley algorithm is processed. The breaking marriage operation (M, m) terminates if the followings occur alternatively:

- w is engaged to m' that w prefers to m,
- some man has been rejected by all women.

It was indicated in [15] that if the matching M' obtained by breaking marriage operation (M, m) terminating with all men engaged, then M' is stable. Furthermore, M' is said to *dominate* M. The term of dominance is defined as follows.

Definition 5 (dominance, see [6])**.** Let M and M' be two stable matchings. M dominates M' (with respect to men) if and only if every man prefers his partner in M at least as well as to his partner in M'.

It was proved that for any two stable matchings M and M', M dominates M' with respect to men, if and only if M' dominates M with respect to women (see [6]). The following result is importance for devising our method which is described in the next section.

Theorem 1 ([15])**.** *Every stable matching M' can be obtained by a series of breaking marriage operations starting from the man-optimal stable matching M_0.*

III. Local Search based Algorithm

Local search is a useful method for solving hard optimization problems. The key idea of local search is to move from a solution to a better one in the domain of feasible solutions of the problem. Searching terminates either when a reasonable solution is found or running time is elapsed. Designing a local search algorithm, usually, needs to deal with three following problems:

- What is an initial solution? This is a feasible point for starting the search.
- For each solution, what is the set of neighbors? This is a local space for the next searching. We also note that the performance of a local search strongly depends on how a neighbor set is constructed.
- Finally, how can we overcome the stuck on locally optimal solution?

Our method iteratively processes through a sequence of stable matchings to find an egalitarian or a sex-equal one. At each iteration, a "good" stable matching will be obtained by means of a local search. Three problems as mentioned above are addressed in the method as follows:

- We initialize a solution of the search to be man-optimal stable matching obtained by Gale-Shapley algorithm.
- For each stable matching, a set of local stable matchings is constructed by breaking marriage which operates on the current stable matching.
- In order to avoid the stuck on locally optimal solution, a small probability is used to randomly choose the next stable matching among ones of the neighbor set.

Our algorithm based on local search is shown in Algorithm 1. The algorithm begins by calling Procedure GaleShapley(I), then assigns to the initial solution as the obtained man-optimal stable matching. At each iteration, a local space for searching, $neighborSet$, is constructed using a sequence of Procedure BreakMarriages(M_{curr}, m) operating on M_{curr} for each m of I (lines 6–10). If $neighborSet$ is empty, meaning that no further neighbors are found, then the algorithm terminates and gives the best-so-far stable matching. Otherwise, the best one among all stable matchings of $neighborSet$ is chosen for the next iteration by means a cost function $f(M)$. By Definition 2, $f(M) = sm(M) + sw(M)$ ($|sm(M) - sw(M)|$, respectively) for finding a egalitarian (sex-equal, respectively) stable matching, where $sm(M)$ and $sw(M)$ are given by (1) and (2), respectively. The algorithm also can randomly choose a stable matching for avoiding the stuck on a local solution (line 16).

By Theorem 1, any stable matching can be obtained from the man-optimal one by successive breaking marriage operations. Algorithm 1 aims to use this property to "walk" through an egalitarian or sex-equal stable matching from the man-optimal one. Because of dominance by the breaking marriage operation, the algorithm will terminates after a finite number of iterations. Particularly, an woman-optimal stable matching should be generated by some breaking marriage operation.

Algorithm 1 Local Search based Algorithm for SMT

```
1: Input: an instance $I$ of SMT, and a positive value $p_0 < 1$.
2: Output: an "optimal" stable matching.

3: $M_{curr}$ := GaleShapley($I$);        ▷ curr stands for current
4: $M_{best} := M_{curr}$;
5: loop
6:     $neighborSet := \emptyset$;
7:     for each man $m$ of $I$ do
8:         $stableMatching$ := BreakMarriages($M_{curr}, m$);
9:         $neighborSet := neighborSet \cup stableMatching$;
10:    end for
11:    if $neighborSet = \emptyset$ then
12:        return $M_{best}$;
13:    end if
14:    $p$ := random probability;
15:    if $p < p_0$ then
16:        $M_{next}$ := a randomly stable matching in $neighborSet$;
17:    else
18:        $M_{next} := \arg\min_{M \in neighborSet} \big(f(M)\big)$;
19:    end if
20:    if $f(M_{best}) > f(M_{next})$ then
21:        $M_{best} := M_{next}$;
22:    end if
23:    $M_{curr} := M_{next}$;
24: end loop

25: procedure GaleShapley($I$)
26:     return the man-optimal stable matching of $I$;
27: end procedure

28: procedure BreakMarriages($M, m$)
29:     return a stable matching obtained from $M$ by breaking $m$;
30: end procedure
```

When the current stable matching is woman-optimal, its neighbor set is empty.

IV. Experimental Results

Random Instance Generation

For SMT problems, we use the method given in [5] to generate the instances. Due to the method, we take two parameters: the problem's size n and a probability, say p_t, of ties. Given a 2-tuple $\langle n, p_t \rangle$, an instance of SMT is iteratively generated as follows:

1) A random preference list of size n for each man and woman is produced.
2) We iterate over each person's (men and women's) preference list: for a man m_i and for his choices c_i from his second to his last, a random value p such that $0 \le p < 1$, is generated; if $p \le p_t$ then the preference for his c_i^{th} choice is the same as c_{i-1}^{th} choice.

An instance generated as $\langle n, 0.0 \rangle$ will be a classical SMP. Table I shows an instance of a SMT which is randomly generated with $\langle 8, 0.5 \rangle$ (ties are denoted by braces).

Simulations

Our algorithm is implemented in Matlab 2012b and run on the platform OS X, Core i5 2.5GHz with 8GB RAM. Fig. 1

TABLE I
AN EXAMPLE OF A SMT RANDOMLY GENERATED WITH $\langle 8, 0.5\rangle$.

Men's list	Women's list
1: (5 7) (1 2) 6 (8 4) 3	1: 5 3 7 (6 1 2 8) 4
2: 2 (3 7) 5 4 1 8 6	2: 8 6 (3 5 7 2 1 4)
3: (8 5 1) 4 6 2 3 7	3: (1 5 6) (2 4) 8 7 3
4: (3 7 2) 4 1 (6 8 5)	4: 8 (7 3 2) (4 1 5) 6
5: (7 2) 5 1 3 (6 8 4)	5: (6 4 7 3 8) (1 2) 5
6: (1 6) (7 5) 8 (4 2 3)	6: 2 8 (5 4) (6 3) 7 1
7: (2 5) 7 6 3 4 (8 1)	7: (7 5 2) 1 8 (6 4) 3
8: (3 8 4 5) (7 2 6 1)	8: (7 4 1) 5 2 3 6 8

shows the behavior of Algorithm 1 for an instance of SMT. The instance is randomly generated with $\langle 100, 0.5\rangle$.

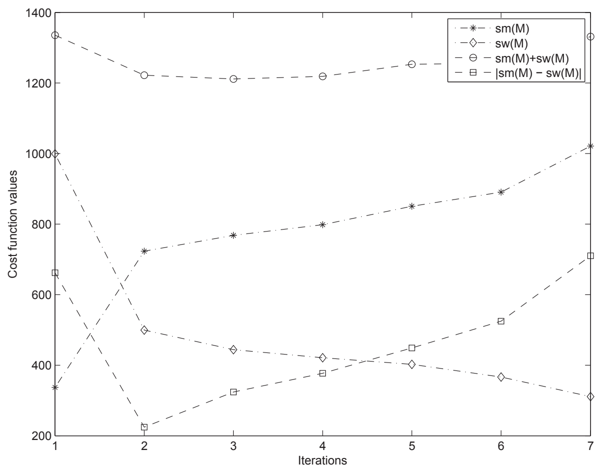

Fig. 1. The behavior of Algorithm 1 testing for a instance randomly generated with $\langle 100, 0.5\rangle$. Because of dominance with respect to men, $sm(M)$ is monotone increasing, meanwhile, $sw(M)$ is monotone decreasing during the process of the algorithm.

TABLE II
TESTING RESULTS OF THE ALGORITHM FOR SMT INSTANCES OF DIFFERENT SIZES. THE NUMBER OF TESTINGS FOR EACH INSTANCE SIZE, $k = 10$.

Size (n)	egalitarian cost		sex-equal cost	
	Mean	Deviation	Mean	Deviation
8	45	0	3	0
16	93	0	13	0
20	138	0	2	0
30	246	0	7	0
50	561	0	56	0
100	1688	0	249	0

In order to show the correctness of our algorithm, we run it for some data sets of different sizes. For each data set of size n, the algorithm is tested for k random instances. The value of p_t is used for all testing to be 0.2. A deviation value of cost functions for instances of size n is computed to evaluate the accuracy of obtained solutions. The deviation values shown in Table II are zeros, meaning that the algorithm successfully finds the egalitarian and sex-equal stable matchings.

Fig. 2 and Fig. 3 show the behaviors of the algorithm for finding, respectively, the egalitarian and sex-equal stable matchings. We run the algorithm for random SMT instances of size 50, varying p_t. For the egalitarian one of each instance of probability p_t, the difference of costs of stable matchings found during the iteration of the algorithm is not too large when compared with that of sex-equal one. This is because of the functions used for evaluating the costs. As mentioned, $f(M) = sm(M) + sw(M)$ ($|sm(M) - sw(M)|$, respectively) is the cost function for searching the egalitarian (sex-equal, respectively) stable matching. By the dominance and weak stability, $sm(M)$ is strictly increasing, meanwhile $sw(M)$ is strictly decreasing. $|sm(M) - sw(M)|$ is thus quite different among stable matchings for each p_t. Furthermore, it also follows that the stable matching found for sex-equal is global. It should be noted that, as shown in both figures, when $p_t = 1.0$ the iteration number for finding both egalitarian and sex-equal stable matchings is one, since the preferences of everybody to opposite-sex one are equal.

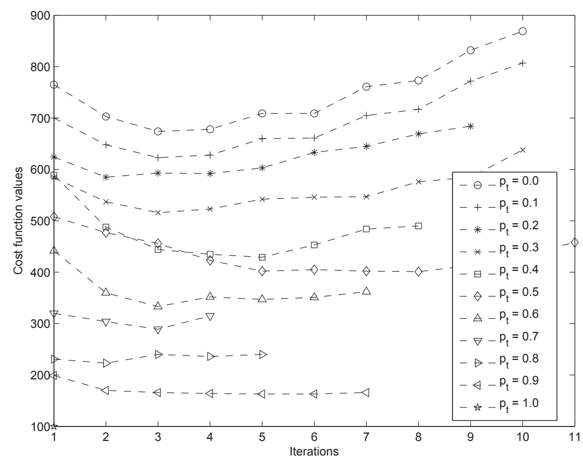

Fig. 2. The behavior for finding egalitarian stable matchings for $\langle 50, p_t\rangle$.

We now discuss the costs of egalitarian and sex-equal stable matchings found by Algorithm 1 for varying p_t. Fig. 4 shows the costs of egalitarian stable matchings obtained for $\langle n, p_t\rangle$, where $n = 8, 10, 20, 30, 50$. For each value of n, we vary p_t from 0 to 1 with step 0.01. The algorithm thus run totally for 505 random instances. We observe that the cost of obtained egalitarian stable matchings decreases as p_t increases. This can be explained as when p_t increases, the number of people have the same rank in preference list also increases. This means that the number of people have high rank decreases as p_t increases. Hence, for each value of n, the value of cost function $sm(M) + sw(M)$ for searching egalitarian stable matchings, decreases as p_t increases. This, however, does not hold for sex-equal stable matchings, since the cost function $|sm(M) - sw(M)|$ is used. Fig. 5 shows the costs of sex-equal matchings for $\langle n, p_t\rangle$, where $n = 8, 30$. Most of cost values is small, because the function $|sm(M) - sw(M)|$ is used for

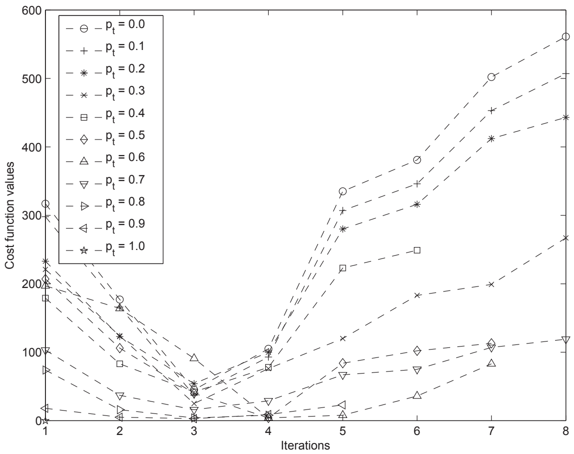

Fig. 3. The behavior for finding sex-equal stable matchings for $\langle 50, p_t \rangle$.

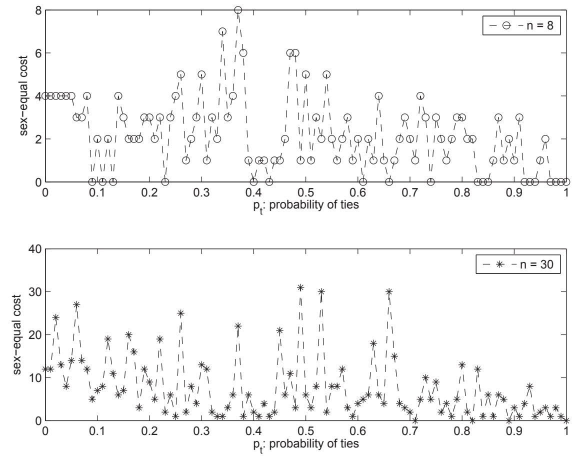

Fig. 5. The cost of finding sex-equal stable matchings for $\langle 8, p_t \rangle$ and $\langle 30, p_t \rangle$.

evaluating the cost of sex-equal stable matchings. The value interval of the cost, in general, is thus smaller when compared with egalitarian ones for each value of n. On the other hand, for $n = 8$, we can observe that there are several obtained matchings whose the same value, while this does not hold for $n = 30$, since the value interval of $|sm(M) - sw(M)|$ is larger than that for $n = 8$.

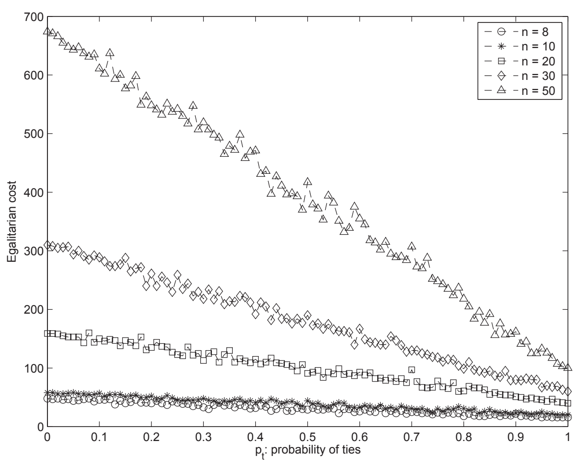

Fig. 4. The cost of finding egalitarian stable matchings for $\langle n, p_t \rangle$.

Finally, we plot the time required for finding egalitarian stable matchings in Fig. 6. We run the algorithm for $\langle n, p_t \rangle$, where $n = 8, 10, 20, 30, 50$, varying p_t with step 0.1. As shown in the figure, the scale of execution time increases as n increases.

V. Conclusion

The paper investigated a local search approach for a variant of the stable marriage problem which allows ties in the preference lists of men and women. Our approach iteratively

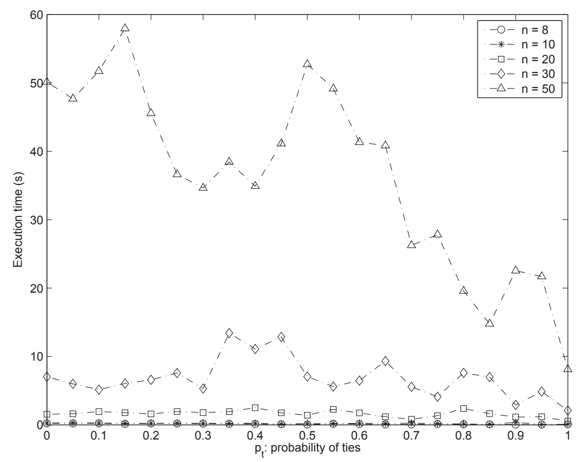

Fig. 6. Execution time of finding egalitarian stable matchings for $\langle n, p_t \rangle$.

searches on a sequence of stable matchings, starting at the man-optimal and ending at woman-optimal one. For each stable matching of the sequence, a set of neighbor stable matchings of the matching is constructed by the breaking marriage operation. A best matching, by means of a cost function, in the set of neighbor is chosen to be the next matching in the sequence. The "optimal", that is egalitarian or sex-equal, is one whose the best value of the cost function so far.

The algorithm processes through a sequence of stable matchings starting at the man-optimal matching and ending at the woman-optimal one. If the solution of an instance is not closed to the woman-optimal matching, a bidirectional search would be appropriate to quickly obtain the solution. In this work, we only consider SMP with ties. Our approach, however, can adapt with some modifications for solving another variant of SMP called SMTI (stable marriage problem with ties and incomplete) which is more practical. These are the topics of future works.

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