AN EFFICIENT ALGORITHM TO FIND A MAXIMUM WEAKLY STABLE MATCHING FOR SPA-ST PROBLEM

Nguyen Thi Uyen[1]✉, **Tran Xuan Sang**[2]

(1) School of Engineering and Technology, Vinh University, Vietnam
(2) Cyber School, Vinh University, Vietnam
{uyennt,sangtx}@vinhuni.edu.vn

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Introduction(1) - SPA-ST problem

- The **S**tudent-**P**roject **A**llocation problem with lecturer preferences over **S**tudents containing **T**ies (SPA-ST) is an extension of the Student-Project Allocation problem (SPA).
- The SPA-ST consists of a set of Students ($\mathcal{S}$), a set of Projects ($\mathcal{P}$), and a set of Lecturers ($\mathcal{L}$). Each lecturer has a capacity d_k. Each project is offered by one lecturer and has a capacity c_j. ($c_j, d_k \in \mathbb{Z}^+$).

Table 1. An instance of SPA-ST

Student's preferences	Lecturer's preferences	
s_1: $(p_1\ p_7)$	l_1: $(s_7\ s_4)\ s_1\ s_3\ (s_2\ s_5)\ s_6$	l_1 offers p_1, p_2, p_3
s_2: $p_1\ p_2\ (p_3\ p_4)\ p_5\ p_6$	l_2: $s_3\ (s_2\ s_6)\ s_7\ s_5$	l_2 offers p_4, p_5, p_6
s_3: $(p_2\ p_1)\ p_4$	l_3: $(s_1\ s_7)$	l_3 offers p_7, p_8
s_4: p_2		
s_5: $(p_1\ p_2)\ p_3\ p_4$		
s_6: $(p_2\ p_3)\ p_4\ p_5\ p_6$	Project capacities	$c_1 = 2$, $c_i = 1$, $(2 \leq i \leq 8)$
s_7: $(p_5\ p_3)\ p_8$	Lecturer capacities	$d_1 = 3$, $d_2 = 2$, $d_3 = 2$

Introduction(2) - Definitions

- A matching M of a $\mathrm{SPA\text{-}ST}$ instance is a set of acceptable pairs (s_i, p_j) or $(s_i, \varnothing)$ such that $|M(s_i)| \leq 1$ for all $s_i \in \mathcal{S}$, $|M(p_j)| \leq c_j$ for all $p_j \in \mathcal{P}$, and $|M(l_k)| \leq d_k$ for all $l_k \in \mathcal{L}$.
- A project p_j is *under-subscribed*, *full* or *over-subscribed* according as $|M(p_j)| < c_j$, $|M(p_j)| = c_j$, or $|M(p_j)| > c_j$, respectively.
- A lecturer l_k is *under-subscribed*, *full* or *over-subscribed* according as $|M(l_k)| < d_k$, $|M(l_k)| = d_k$, or $|M(l_k)| > d_k$, respectively.
- If $(s_i, p_j) \in M$, then s_i is matched to p_j, denoted by $M(s_i) = p_j$. If $M(s_i) = \varnothing$, then s_i is *unmatched* in M.
- Example
  $M = \{(s_1, p_1), (s_2, \varnothing), (s_3, p_4), (s_4, p_2), (s_5, \varnothing), (s_6, p_5), (s_7, p_3)\}$, where s_2, s_5 are unmatched, p_1 is *under-subscribed*, and l_1 is *full*.

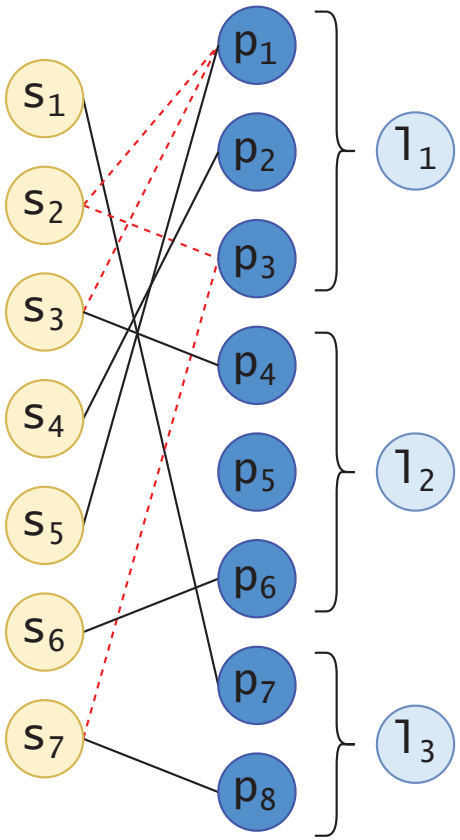
Introduction(3) - Definitions

Let $(s_i, p_j) \in (\mathcal{S} \times \mathcal{P}) \setminus M$ be a blocking pair for a weakly stable matching M if the following conditions are satisfied:

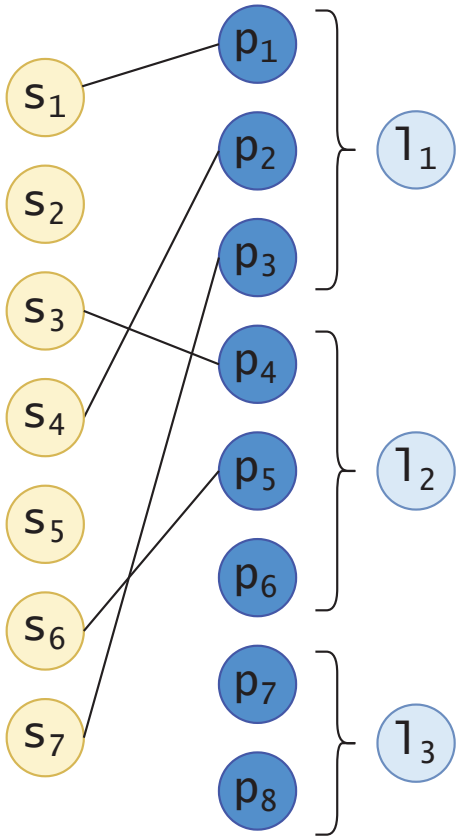
1. s_i and p_j find accept each other;
2. s_i prefers p_j to $M(s_i)$ or $M(s_i) = \varnothing$;
3. either (a), (b) or (c) holds as follows:
   - $|M(p_j)| < c_j$ and $|M(l_k)| < d_k$;
   - $|M(p_j)| < c_j$, $|M(l_k)| = d_k$ and;
     - either $s_i \in M(l_k)$ or;
     - l_k prefers s_i to the worst student in $M(l_k)$;
   - $|M(p_j)| = c_j$ and l_k prefers s_i to the worst student in $M(p_j)$.

- A matching M is called *weakly stable* if it admits no blocking pair, otherwise it is called *unstable*. $|M|$ is called the number of matched students in M. If $|M| =$ n, then M is a *perfect* matching, otherwise, M is a *non-perfect* matching.

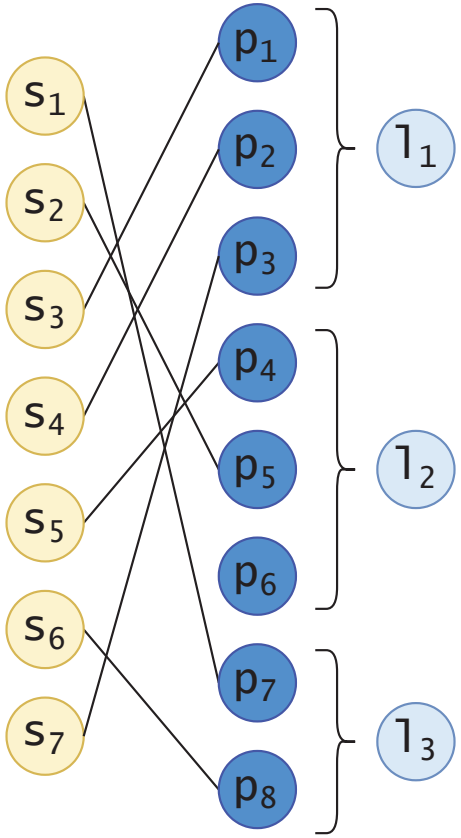
Introduction(4) - Definitions



M is unstable



M is stable



M is perfect

Related works

- Several researchers have focused on solving the $\mathrm{SPA\text{-}ST}$ problem because of its applications at the Glasgow University [Kwanashie et al.2014], Southern Denmark University [Chiarandini et al.2019], York University [Kazakov et al.2002].
- Cooper et al.2018 presented a $3/2$- approximation algorithm, called AP, to find a *weakly stable* matching
- Besides, they also modeled the $\mathrm{SPA\text{-}ST}$ problem as an Integer Programming (IP) problem.
- Olaosebikan et al.2020 described the polynomial-time algorithm to find a *strongly stable* matching.
- Olaosebikan et al.2020 proposed an approximation algorithm for solving $\mathrm{SPA\text{-}ST}$ problem in terms of finding a *super-stable* matching.

Proposed Algorithm(1) - Main idea

- This paper presents an effective heuristic algorithm to solve the $\mathrm{MAX\text{-}SPA\text{-}ST}$ problem of large sizes.
- Our main idea is to start from a stable matching, then define two heuristic strategies promoting *unmatched students* and *under-subscribed* lecturers to improve the matching size by breaking stable pairs.
- Our algorithm terminates when it finds a *perfect matching* or reaches a maximum number of iterations.
- The experimental results show that our proposed algorithm is more efficient than the AP (Cooper et al.2018) algorithm in terms of solution quality and execution time.

Proposed Algorithm(2) - Pseudocode

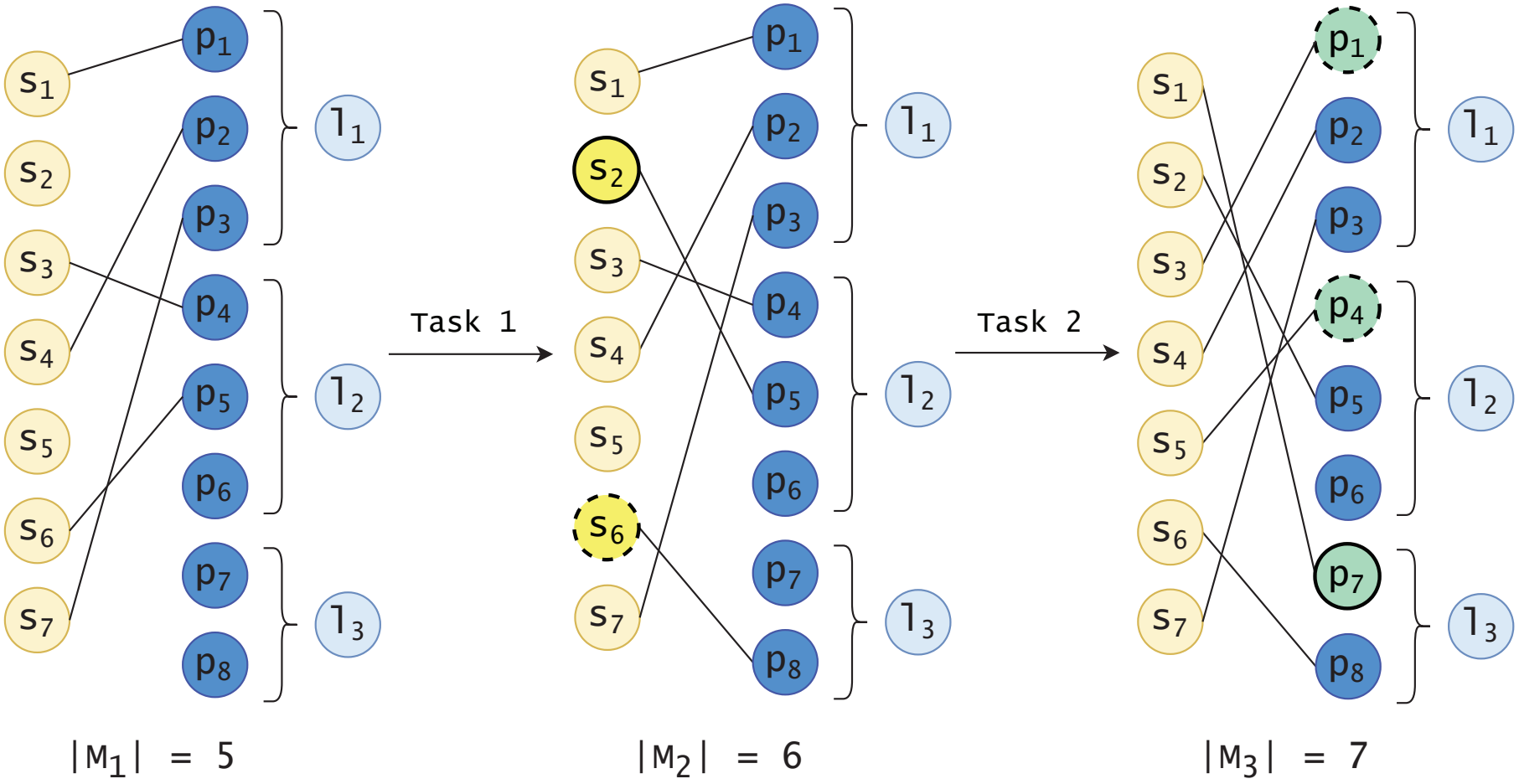
Algorithm 1: HA Algorithm for MAX-SPA-ST problem

Input: - An SPA-ST instance I.
- max_iter is the maximum number of iterations.

Output: A maximum stable matching M.

```
function HA(I)
    M := EGS(I);                                   ▷ generate a stable matching
    v(s_i) := 0, (1 ≤ i ≤ n);                      ▷ mark the replacing time of s_i
    v(p_i) := 0, (1 ≤ i ≤ q);                      ▷ mark the replacing time of p_i
    iter := 0;
    while (iter ≤ max_iter) do
        iter := iter + 1;
        if |M| = n then break ;
        s_i := a random unmatched student in M;                    ▷ Task 1
        R'_{s_i} := s_i's ranks list;
        while R'_{s_i} is non-empty do
            p_j := argmin(R'_{s_i} > 0), ∀p_j ∈ P;
            l_k := a lecturer who offers p_j;
            for (each s_t ∈ M(l_k)| R_{l_k}(s_t) = R_{l_k}(s_i)) do
                if (|M(p_j)| < c_j) or (s_t ∈ M(p_j) and |M(p_j)| = c_j) then
                    if v(s_i) ≥ v(s_t) or a small probability then
                        M := M \ {(s_t, p_z)} ∪ {(s_i, p_j)}| p_z = M(s_t);
                        v(s_i) := v(s_i) + 1;
                        Repair(p_z, l_k);
                        M := Break_Student(M, s_t);
                        break;
            if M(s_i) ≠ ∅ then break;
            else R'_{s_i}(p_j) := 0 ;
        p_i := a random under-subscribed project in M;             ▷ Task 2
        l_k := a lecturer who offers p_i;
        for each s_j ∈ S|R_{s_j}(p_t) = R_{s_j}(p_i)|p_t = M(s_j) do
            if (|M(l_k)| < d_k) or (|M(l_k)| = d_k and s_j ∈ M(l_k)) then
                if (v(p_i) ≥ v(p_t) or a small probability then
                    M := M \ {(s_j, p_t)} ∪ {(s_j, p_i)};
                    v(p_i) := v(p_i) + 1;
                    M := Break_Lecturer(M, p_t);
                    break;
    return M;
end function
```

Proposed Algorithm(3) - Example



Experiment results

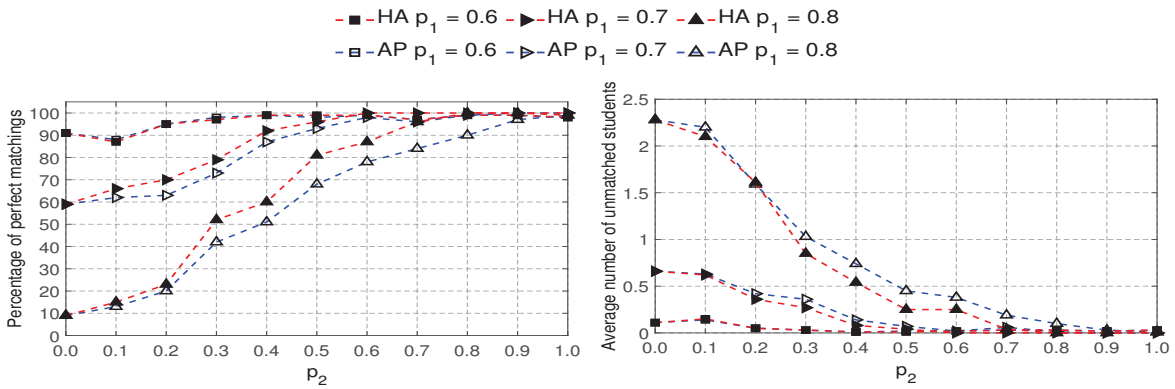
Dataset

- In this section, we compared the solution quality and execution time of HA with those of AP which is an approximation algorithm (Cooper at al.2018) for the $\mathrm{MAX\text{-}SPA\text{-}ST}$ problem. We implemented these algorithms by Matlab R2019a software on a Xeon-R Gold 6130 CPU 2.1 GHz computer with 16 GB RAM.
- To perform the experiments, we generated randomly $\mathrm{SPA\text{-}ST}$ instances with five parameters (n, m, q, p_1, p_2), where n is the number of students, m is the number of lecturers, q is the number of projects, p_1 is the probability of incompleteness, and p_2 is the probability of ties. By this setting, on average, each student ranks about $q \times (1 - p_1)$ projects. In our experiments, we set the total capacity of projects and lecturers as $C = 1.2n$ and $D = 1.1n$, respectively.

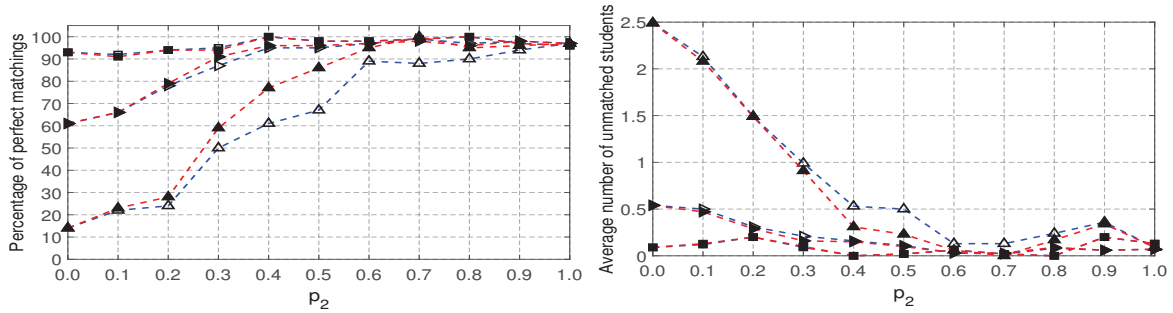
Experiment results

Comparison of solution quality

- **Experiment 1**: Firstly, we randomly generated 100 instances of SPA-ST for parameters (n, m, q, p_1, p_2) with $n \in \{100, 200\}$, $m = 0.05n$, $q = 0.1n$, $p_1 \in [0.1, 0.8]$ with step 0.1, and $p_2 \in [0.0, 1.0]$ with step 0.1.



(a) – Solution quality for $n = 100$, $m = 5$, $q = 10$ – (b)

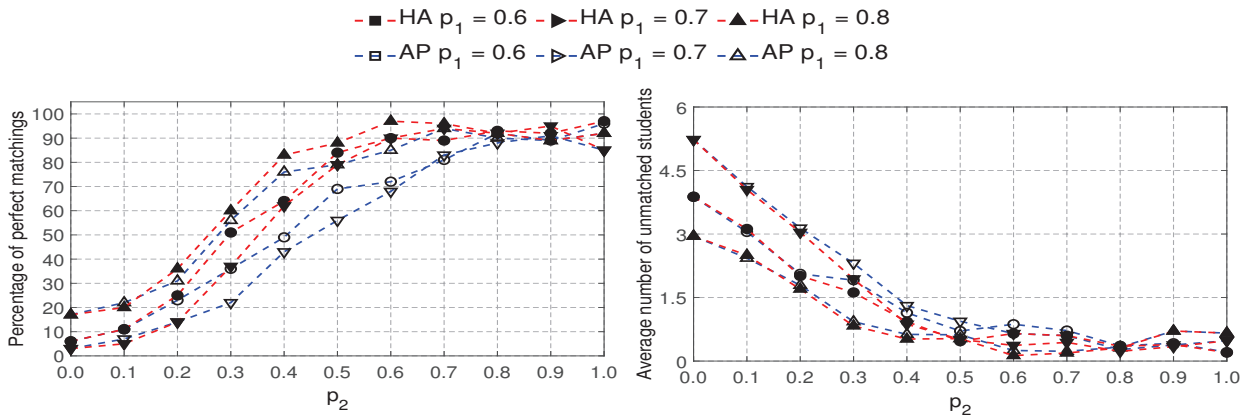


(c) – Solution quality for $n = 200$, $m = 10$, $q = 20$ – (d)

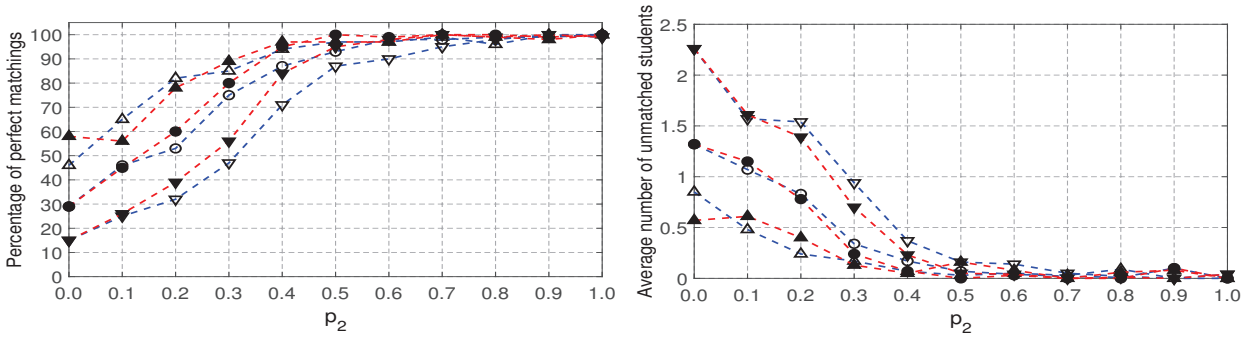
Experiment results

Comparison of solution quality

- **Experiment 2**: In this experiment, we changed $n \in \{300, 400\}$, $p_1 \in \{0.82, 0.84, 0.86\}$ and kept the values of m, q, and p_2 as in Experiment 1



(a) – Solution quality for $n = 300$, $m = 15$, $q = 30$ – (b)



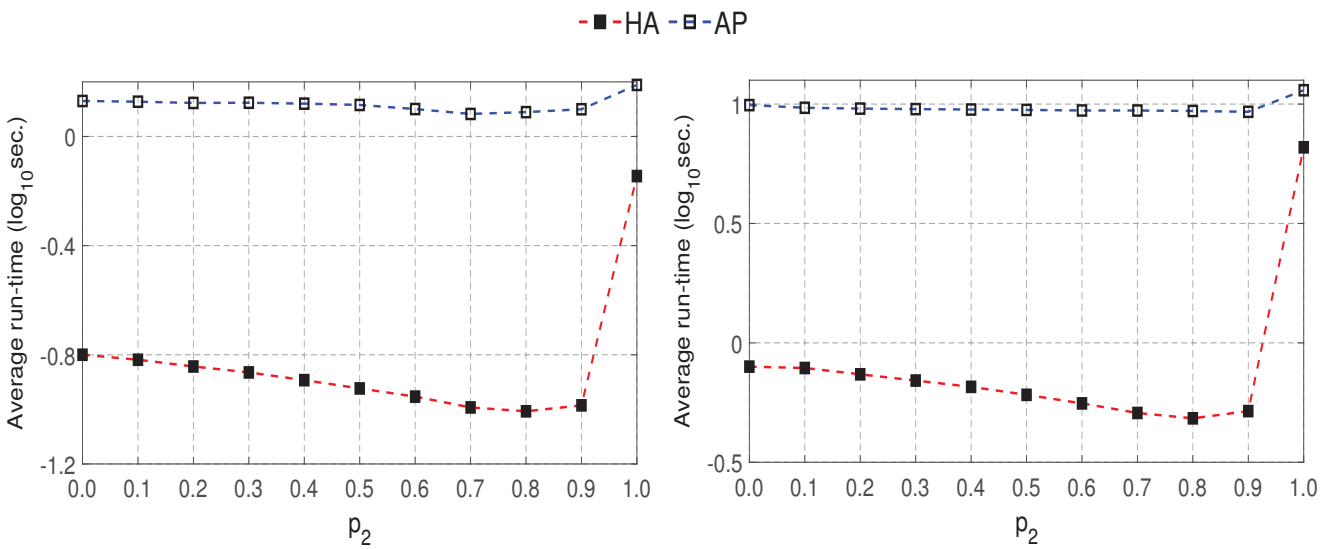
(c) – Solution quality for $n = 400$, $m = 20$, $q = 40$ – (d)

Fig. 2. Percentage of perfect matching and average number of unmatched students

Experiment results

Comparison of execution time

- **Experiment 3**: We randomly generated 100 instances of SPA-ST for parameters (n, m, q, p_1, p_2) with $n \in \{1000, 2000\}$, $m = 0.05n$, $q = 0.4n$, $p_1 \in [0.1, 0.8]$ with step 0.1, and $p_2 \in [0.0, 1.0]$ with step 0.1

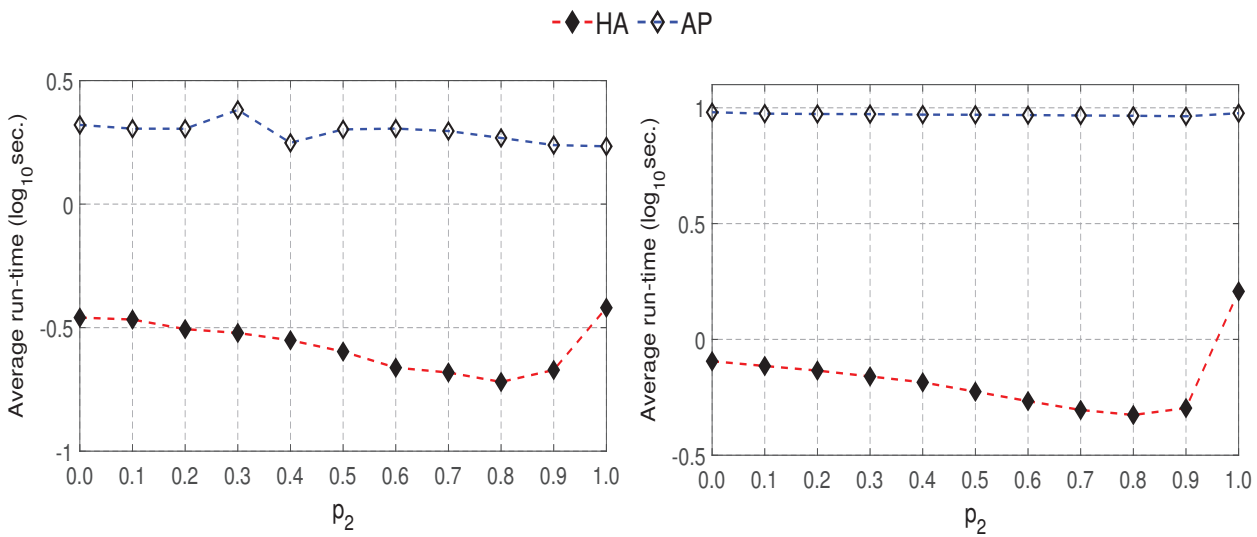


(a) $n = 1000, p_1 \in [0.1, 0.8]$ (b) $n = 2000, p_1 \in [0.1, 0.8]$

Experiment results

Comparison of execution time

- **Experiment 4**: Finally, we kept the values of n, m, q, and p_2 as in Experiment 3, increased the values of $p_1 \in [0.81, 0.89]$ with step 0.01, and randomly generated 100 instances of $\mathrm{SPA\text{-}ST}$ for each combination of values (p_1, p_2). By increasing the values of p_1, we aim to reduce the number of projects ranked by each student compared to Experiment 3.



(c) $n = 1000, p_1 \in [0.81, 0.89]$ (d) $n = 2000, p_1 \in [0.81, 0.89]$

Conclusion

- In this study, we presented a heuristic algorithm for solving the $\mathrm{MAX\text{-}SPA\text{-}ST}$ problem.
- We started with a stable matching and improved the matching size by defining two heuristic strategies to pair the *unmatched* students and *under-subscribed* projects.
- The experimental results showed that our proposed algorithm is efficient in terms of solution quality and execution time for the $\mathrm{MAX\text{-}SPA\text{-}ST}$ problem of large sizes.
- In the future, we will extend this proposed approach to solve the other variants of the SPA problem.

Thank for listening!

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