ABILITY TO EMIT STIMULATED TERAHERTZ RADIATION BETWEEN LANDAU LEVELS IN GaAs/Al GaAs QUANTUM WELL

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Abstract: In this paper we show that a population inversion can be achieved in the system of Landau levels in quantum well structures by a considerable difference between intersubband and intrasubband electron-electron scattering rate. This mechanism allows the generation of emission terahertz stimulated radiation. The frequency of the radiation may continuously be tuned in a wide range of terahertz frequencies by the variation of the magnetic field strength.
Keywords: Quantum well; Landau level; electron-electron scattering; population inversion.

1. Introduction

It was shown that in quantum wells with energy spacing between 1^{st} and 2^{nd} subbands lower than an optical phonon energy (i.e. when optical phonon scattering is suppressed) the population of zeroth Landau level (LL) in the 2^{nd} subband exceeds that of the 1^{st} LL in the 1^{st} subband [1-3]. In [4], it was shown that with increasing the electron concentration electrons in quantum wells the intensity of the electron-electron scattering increases much faster than that of the single electron scattering. Then electron-electron scattering was considered as the main mechanism for redistributing electron density according to the LLs.

The main problem remaining is that the (2,0)→(1,1) optical transition is forbidden by selection rules when magnetic field B is orthogonal to structure layers [6]. An investigation on the effect of a tilted magnetic field sloping on the optical matrix element of intersubband transitions was indicated in [5]. The importance of an asymmetric structure for achieving significant values of transition dipole matrix elements was revealed, and an asymmetric two-well structure was proposed as a possible solution. Asymmetric potential profile depends on the widths of the wells, which allow maximization of the optical matrix element of the optical intersubband transitions, which is of interest. Therefore, a stimulated emission of terahertz radiation should be achieved on radiative transitions between these LLs.

In the present paper, we report a possibility to achieve a population inversion between the LLs (2,0) and (1,1) in cascade quantum well structures in a strong magnetic field under a condition of sequential resonant tunneling. The scheme of transitions between these Landau levels is shown in Fig.1. The population inversion is caused by a considerable difference between intersubband and intrasubband scattering times. Upper LL (2,0) is depopulated by intersubband scattering only, while the lower LL (1,1) - by intra-subband scattering to the zeroth level of the same subband. In the considered system, the electron-electron scattering is the most important inter-LL scattering mechanism, since other inter-LL scattering processed are strongly suppressed [3, 4]. So,

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the main attention was paid to developing a model of electron-electron scattering in a system of LLs in quantum well with several subbands. In order to provide a nonzero dipole matrix element for transitions of interest along with the application of the tilted magnetic field, it is necessary to introduce an asymmetric potential profile along the direction of the structure growth. In this paper we proposed application of transverse electric field to achieved asymmetric potential along the growth axis of the structure. The proposed mechanism allows achieve a continuous tunable frequency stimulated emission because of energy spacing between LLs (2,0) and (1,1) can be varied by magnetic field strength.

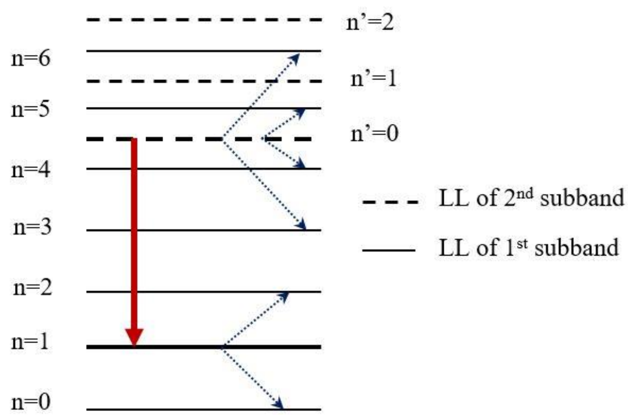

Figure 1: *Proposed scheme of transitions between Landau levels in a quantum well. The thick-solid-arrow indicates the (2,0)→(1,1) optical transition, and the dotted arrows mark the transitions due to the electron-electron scattering.*

2. Theoretical background

Consider the electron kinetics in sequential resonant tunneling GaAs/AlGaAs quantum well structures in quantizing magnetic field $\mathbf{B} = B\mathbf{e}_z$ (axis z is perpendicular to the layers). In Landau gauge the single-electron spectrum is a set of confinement subbands each splitting into a series of Landau levels [5].

$$E_{(\nu,n)} = \varepsilon_{(\nu,n)} + \hbar\omega_C(n+\tfrac{1}{2}) \tag{1}$$

with wave functions

$$\psi(x,y,z) = \frac{\exp(ikx)}{\sqrt{L}}\varphi_\nu(\mathrm{z})\Phi_n(y-k\ell_\perp^2) \tag{2}$$

determined by three quantum numbers - subband index ν, Landau level number *n* and x-axis momentum component *k*. $\ell_\perp = \sqrt{\frac{\hbar c}{eB}}$ is the magnetic length. $\Phi_n(y)$ is the wave function of harmonic oscillator with effective mass m and cyclotron frequency $\omega_C = \frac{eB}{mc}$. $\varepsilon_{(\nu,n)}$ and $\varphi_\nu(z)$ are the energy and wave function of ν th subband.

Using the Fermi golden rule approximation, the electron flux to Landau state $i=(\nu_i, n_i)$ caused by electron-electron scattering transitions $\left(i=(\nu_i, n_i), j=(\nu_j, n_j)\right) \rightarrow \left(f=(\nu_f, n_f), j=(\nu_g, n_g)\right)$ is given by the expression

$$j_{(i,j)\to(f,g)} = \frac{N_i}{\tau_{(i,j)\to(f,g)}} . \tag{3}$$

Where

$$\frac{1}{\tau_{(i,j)\to(f,g)}} = \frac{2}{L^2\alpha}\sum_{k_i}\frac{1}{\tilde{\tau}_{(i,j)\to(f,g)}(k_i)} \tag{4}$$

is total $(i,j)\to(f,g)$ transition rate, $\tau_{(i,j)\to(f,g)}$ is lifetime of electron on LL (*i*,*j*).

$$\frac{1}{\tilde{\tau}_{(i,j)\to(f,g)}(k_i)} = \frac{2\pi}{\hbar}\sum_{k_j,k_f,k_g}\left|V_{(i,j)\to(f,g)}(k_i,k_j,k_f,k_g)\right|^2 \frac{N_j}{\alpha}\left[1-\frac{N_f}{\alpha}\right]\left[1-\frac{N_g}{\alpha}\right]\delta(E_i+E_j-E_f-E_g) \tag{5}$$

is scattering rate from initial state containing one electron in LL i with wave vector k_i and one electron in LL j to the final states containing electrons in LLs f and g, $\alpha = \frac{1}{\pi\ell_\perp^2}$ - LL degeneracy, N - 2D electron concentration on the corresponding Landau state.

$$V_{(i,j)\to(f,g)}(k_i,k_j,k_f,k_g) = \int d\mathbf{r}_1 d\mathbf{r}_2 \psi_f^*(\mathbf{r}_1,k_f)\psi_i(\mathbf{r}_1,k_i)\frac{e^2}{\varepsilon_s\left|\mathbf{r}_1-\mathbf{r}_2\right|^2}\psi_g^*(\mathbf{r}_2,k_g)\psi_j(\mathbf{r}_2,k_j) \tag{6}$$

is matrix elements of electron-electron interaction.

The finite width of the LLs is taken into account by replacing the δ-function in equation (5) by a form-factor $F_{(i,j)\to(f,g)}(E_i+E_j-E_f-E_g)$, which is approximated by Lorentzian

$$F_{(i,j)\to(f,g)}(E) = \frac{1}{\pi}\frac{\Gamma}{E^2+\Gamma^2}, \tag{7}$$

halfwidth Γ = 2 meV, typical values for quantum well structures [3]

3. Results of the numerical calculation and discussion

The results of the numerical calculation of the e-e scattering times (5) are shown in Fig. 2. The calculations were carried out for GaAs/$Al_{0.3}Ga_{0.7}As$ quantum wells of 25 nm width, with intersubband spacing $\Delta E_{12} = 20.4$ meV. The lifetime of the lower (1,1) LL was found to depend monotonically and approximately linearly upon magnetic field. On the contrary, the intersubband scattering lifetime of LL (2,0) revealed an oscillating dependence on magnetic field. The minima of lifetimes correspond to the situation when LL (2,0) coincides with certain LL of lower subband or is situated exactly between them.

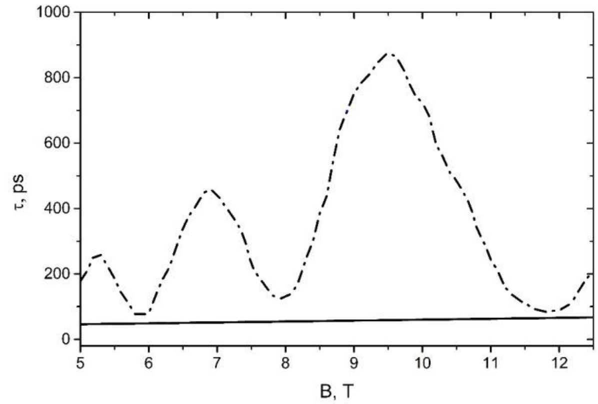

Figure 2: *The numerical calculated e-e scattering lifetimes for LL (2,0) (dash-dot line) and LL (1,1) (solid line) Landau levels in GaAs/Al$_{0.3}$Ga$_{0.7}$As quantum well of 25 nm width,* $N_{(2,0)} = N_{(1,1)} = 10^{10}\,\mathrm{cm}^{-2}$.

Result of the numerical calculation (Fig. 2) shows that the intrasubband scattering time is considerably shorter than that of intersubband one. Within the range 5÷12 T of magnetic field strength the lifetime of the upper (2,0) LL is always higher than that of the lower (1,1) LL, and in the intervals B = 5.0÷5.6 T, 6.2÷7.6 T, 8.1÷11.0 T the difference achieves to 3-10 times. Then the scattering rate from the LL (1,1) is greater than scattering rate from the LL (2,0). It makes possible to achieve a population inversion for considered inter-LL transition by the resonant tunneling pumping of the upper (2,0) level. The emission frequency

$$\hbar\omega = \Delta E_{12} - \hbar\omega_C \tag{8}$$

will vary within interval $\hbar\omega = 11.8 \div 3.2$ meV (λ= 105÷390 μm, f = 2.86÷0.77 THz) for magnetic field range B = 5 ÷ 10 T.

The important point is that the specified (2,0)→(1,1) optical transition is forbidden by selection rules when magnetic field **B** is orthogonal to structure layers [4]. The problem can be solved by placing the structure into a tilted magnetic field $\mathbf{B} = B_{\|}\mathbf{e}_y + B\mathbf{e}_z$ where the mentioned selection rule is violated due to the mixing of in-plane and out-of-plane carrier motion in resonant tunneling transitions between the LLs in tilted magnetic field. If the cyclotron energy is several times lower than intersubband spacing the following expression for wave function [5, 7]

$$\psi_{\nu,n}(y,z) = \frac{\exp(ikx)}{\sqrt{L}}\varphi_\nu(z)\Phi_n\left(y - \tfrac{\ell_\perp^2}{\ell_\|^2}\langle z\rangle_\nu\right), \tag{9}$$

where $\langle z\rangle_\nu = \int |\varphi_\nu(z)|^2 z dz$ is the average value of the electron coordinate along the z axis in the ν^{th} subband state and $\ell_{\|} = \sqrt{\frac{\hbar c}{eB_{\|}}}$ are the magnetic lengths for longitudinal ($B_\|$) magnetic field component.

Using this wave function, the dipole matrix element can be obtained

$$\left|\mathbf{D}_{(2,0)\to(1,1)}\right|^2 = \langle \psi_f | \mathbf{r} | \psi_i \rangle = \delta_{k_1,k_2} \left| \langle \varphi_2(z) | z | \varphi_1(z) \rangle \right|^2 \frac{\xi^2}{2} \exp\left(-\frac{\xi^2}{2}\right). \quad (10)$$

Here

$$\xi = \left[\langle z \rangle_2 - \langle z \rangle_1 \right] \frac{\ell_\perp}{\ell_\parallel^2}. \quad (11)$$

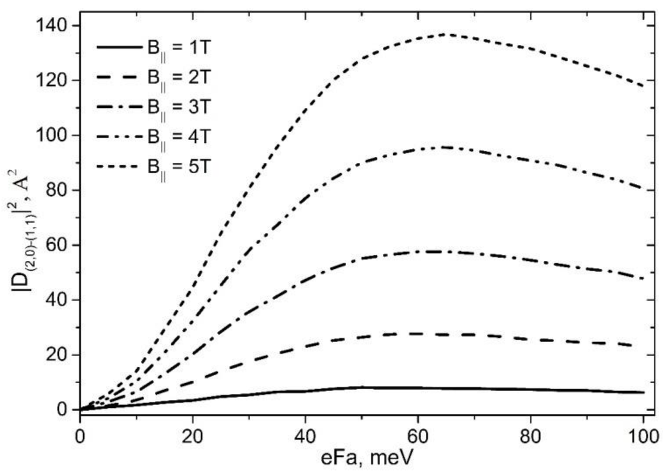

Figure 3: *The numerical calculated dependence of dipole matrix element* $\left|\mathbf{D}_{(2,0)\to(1,1)}\right|^2$ *on the voltage drop per quantum well eFa in tilted magnetic field for different values of the parallel to layers component* $B_{\parallel}$ *= 1÷5 T. The calculations were made for GaAs/*$Al_{0.3}Ga_{0.7}As$ *quantum well of 25 nm width. The perpendicular to layers component of magnetic field B = 5 T.*

We can see that the dipole matrix element becomes nonzero only if the values $\langle z \rangle_2$ and $\langle z \rangle_1$ are substantially different. In quantum well with symmetric potential, the subband wave functions $\varphi_\nu(z)$ are symmetric or antisymmetric with respect to symmetry center of the potential, and the averages $\langle z \rangle_\nu$ are the same for all subbands. So, in symmetric potential, the transition matrix element continues to be close to zero even in the tilted magnetic field. Thus, to provide a nonzero dipole matrix element for transitions of interest along with the application of the tilted magnetic field, it is necessary to introduce an asymmetric potential profile along the direction of the structure growth. It can be achieved by introducing asymmetric potential along the growth axis of the structure. The structure (the width of the quantum wells) considered is an example proposed to illustrate the general way of how the selection rule forbidding the transitions of interest can be overcome [5]. For simplicity, in this paper we propose application of transverse electric field to achieve asymmetric potential along the growth axis of the structure.

In Fig. 3, the numerical calculation according to (10) matrix element dependence on electric field for different tilt angles of magnetic field is shown. It can be seen that the application of the electric field results in the nonzero dipole matrix element. So, the stimulated emission of terahertz radiation can be achieved on the transitions between LLs (2,0) and (1,1), and the emission frequency may continuously be tuned in a wide range of terahertz frequencies by the variation of the magnetic field strength according to the relation (8).

4. Conclusion

It was shown that due to a difference between intersubband and intrasubband e-e scattering rates, a population of the Landau levels of the upper subbands can considerably exceed that of the Landau levels of the lowest one, and a wide-range tunable terahertz stimulated emission on such inter-Landau level transition can be achieved.

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TÓM TẮT

KHẢ NĂNG PHÁT BỨC XẠ KÍCH THÍCH GIỮA CÁC MỨC LANDAU TRONG GIẾNG LƯỢNG TỬ GaAs/AlGaAs

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Trong bài báo này chúng tôi chỉ ra sự nghịch đảo mật độ cư trú giữa các mức Landau trong giếng lượng tử có thể đạt được bởi sự khác biệt đáng kể giữa tốc độ tán xạ electron-electron giữa các vùng con và bên trong một vùng con. Cơ chế này cho phép phát bức xạ kích thích mà tần số của bức xạ đó có thể điều chỉnh được một cách liên tục trong phạm vi rộng của miền tần số THz (miền hồng ngoại) bởi thay đổi cường độ từ trường.

Từ khóa: Giếng lượng tử; mức Landau; tán xạ electron-electron; nghịch đảo mật độ cư trú.