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Propagation of laser pulse in a three-level cascade inhomogeneously broadened medium under electromagnetically induced transparency conditions

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ABSTRACT

We study a pair of probe and coupling laser pulses propagating in a Doppler broadened three-level cascade atomic medium. Influence of the coupling pulse area on the probe pulse shape is studied in a wide region of pulse duration, from micro to pico second. It is found that the needed value of coupling pulse area to establish an electromagnetically induced transparency (EIT) for the probe light, namely the probe pulse shape is unchanged, is smaller at shorter pulse duration. On the other hand, influence of Doppler broadening is more efficient in the long duration side. These results can find interesting applications in all optical switching, quantum information processing, and transmission.

1. Introduction

In the past decades, the propagation of laser pulses in resonant atomic media is an attractive research topic in nonlinear and quantum optics due to its interesting effects and potential applications in quantum engineering and optical communication. A significant interest in this topic focuses on coherent control of the absorption, dispersion, and nonlinearity coefficients under the conditions of EIT [1–4]. The unusual optical properties of the EIT medium have opened several promising subjects as lasing without inversion [5], slow-light group velocity [6], quantum information [7], all optical switching [8], nonlinear optics at low light level [9], and enhancement of Kerr nonlinearity [10,11], optical bistability [12,13] etc.

In addition to the steady regime, dynamics of light pulses propagating in EIT media has also attracted a great attention because of its potential applications. The pioneered works on pulse propagation in a three-level lambda system under the EIT condition were introduced by Eberly [14] and Harris el al. [15]. They predicted energy preparation losses at a front edge of pulse before reaching an EIT regime. Numerous works were lately performed, e.g., propagation of soliton-like pulses in three- and five-level systems [16], adiabatic propagation of short pulses under the EIT condition [17], dynamical control of light pulse propagation [18–20], propagation of coupled ultraslow optical soliton pairs in a cold atomic system [21], controlling photons using EIT [22], EIT with tunable single-photon pulses [23], influence of propagation on the coherent accumulation [24] and high resolution atomic coherent control used spectral phase manipulation of a femto-second optical frequency comb [25].

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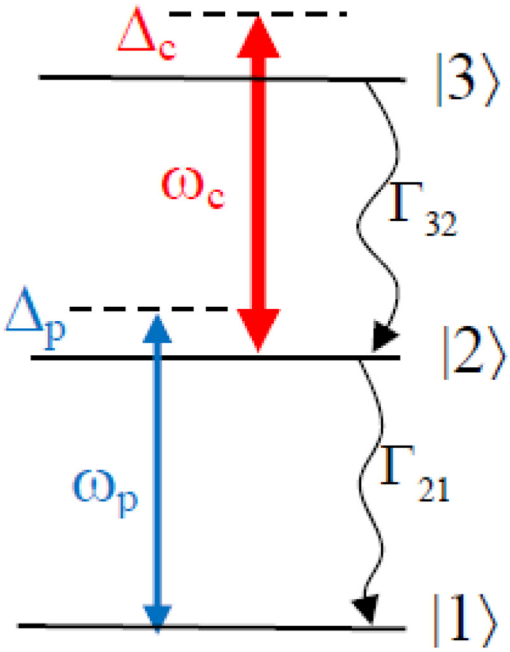

Fig. 1. The three-level cascade scheme excited by the probe and coupling laser fields.

Growing of interest on light propagation is also extended to hot gaseous EIT media due to it closes to normal condition and has interesting effects. An early study of light pulse propagation in inhomogeneously broadened two-level medium was introduced by Wilson-Gordon, et al. [26]. Since then, Doppler broadened media have been considered in various aspects, e.g., influences of linear and nonlinear effects on pulse propagation [27,28], population transfer by single-frequency chirped short laser pulse [29], phase control of light propagation [30,31], pulse compression by coherent control [32].

Recently, T. Nakajima et al., [33] have studied theoretically propagation of two short laser pulse trains in a Λ-type three-level medium under conditions of EIT by means of probability amplitude. Although the obtained result showed interesting phenomena of pulse propagation but there is still lack of Doppler broadening which is important for experimental realizations in the gaseous phase and long pulse duration. In order to bridge this gap, in this work, we study the influence of the Doppler broadening on propagation of a pair of probe and coupling laser pulses in a three-level cascade-type ^{87}Rb atomic medium. Using density matrix formalism, the dynamics of the pulses is represented by the Maxwell–Bloch equations under the electric dipole- and rotating wave- approximations. Influences of the coupling laser and Doppler broadening on the temporal and spatial evolution of the probe laser pulse are investigated in a wide region from micro to pico second of pulse duration.

2. Theoretical model

We consider a gaseous atomic medium excited by a pair of probe and coupling laser pulses via the three-level cascade scheme, as in Fig. 1. The probe (with frequency ω_p and electric field amplitude E_p) and coupling (with frequency ω_c and electric field amplitude E_c) laser pulses drive the allowed electric dipole transitions $|1\rangle \leftrightarrow |2\rangle$ and $|2\rangle \leftrightarrow |3\rangle$, respectively. We denote Γ_{21} and Γ_{32} are the population decay rate of the states $|2\rangle$ and $|3\rangle$, respectively.

Both probe and coupling pulses co-propagate along z-axes through the medium. Each light field is described classically so that the total electric field is written as:

$$\vec{E}(z,t) = \vec{e}_p \mathscr{E}_{p0} f_p(z,t) e^{-i(\omega_p t - k_p z)} + \vec{e}_c \mathscr{E}_{c0} f_c(z,t) e^{-i(\omega_c t - k_c z)} + c.c, \tag{1}$$

where,k_l, $\vec{e}_l$, and $f_l(z,t)$ are the wave number, polarization vector, and pulse envelope of the probe ($l=p$) or coupling ($l=c$) field, respectively. We restrict to the case where both probe and coupling laser pulses have the same temporal width τ_0 with a Gaussian-type envelop at the entrance of the medium, namely

$$f(z=0,t) = e^{-\pi\left(\frac{t}{\tau_0}\right)^2}. \tag{2}$$

In the framework of semi-classical theory, evolution of density matrix operator ρ of the system is represented by the following Liouville equation:

$$\dot{\rho} = -\frac{i}{?}[H,\rho] + \Lambda\rho, \tag{3}$$

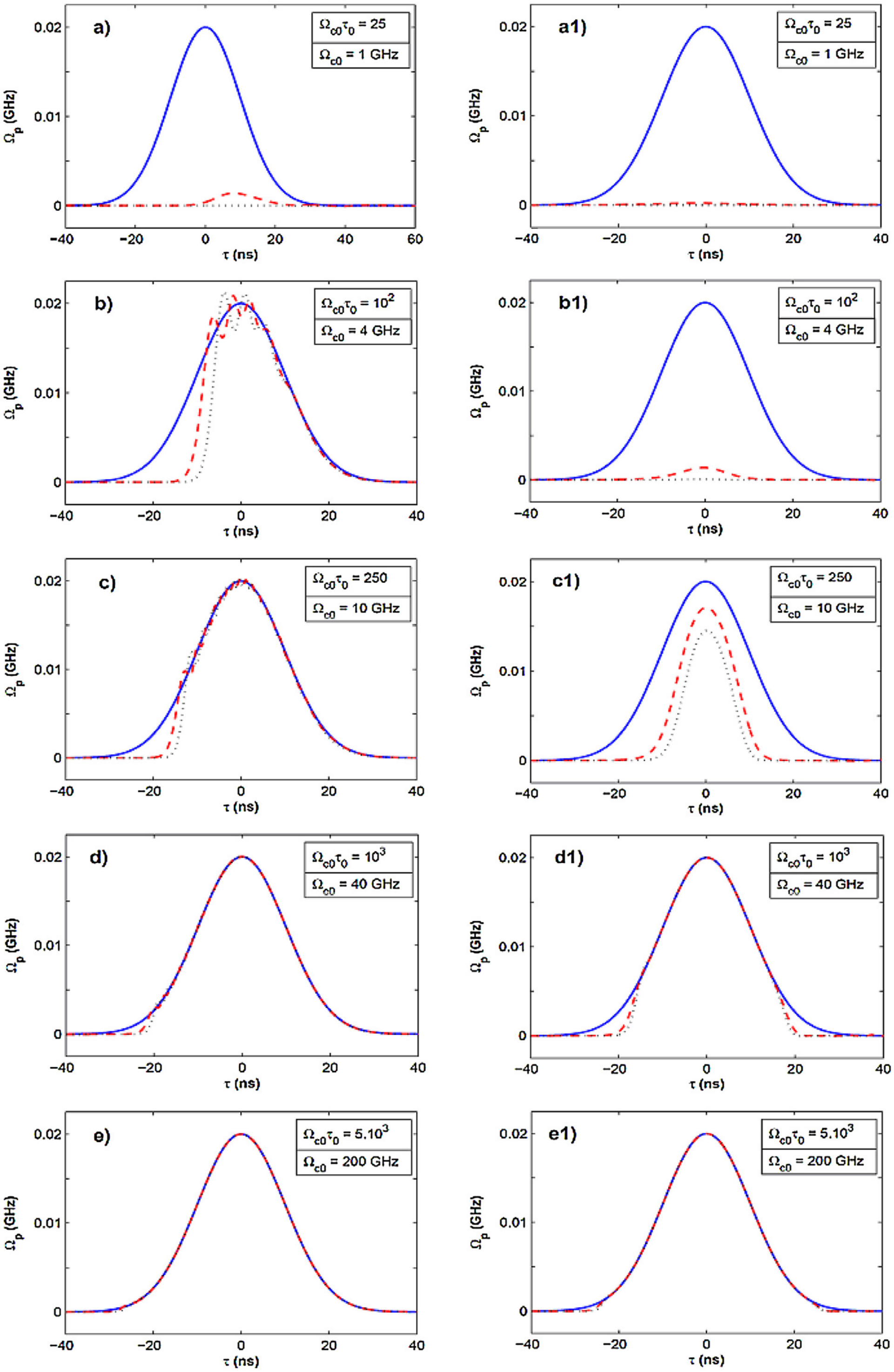

Fig. 2. Temporal evolution of the probe pulse $\Omega_p(\xi,\tau)$ at $\mu_p\xi = 0$ (*solid*), 5 ns^{-1} (*dashed*), and 10 ns^{-1} (*dotted*) when $\tau_0 = 25$ ns in the absence (*left*) and presence (*right*) of the Doppler broadening, respectively.

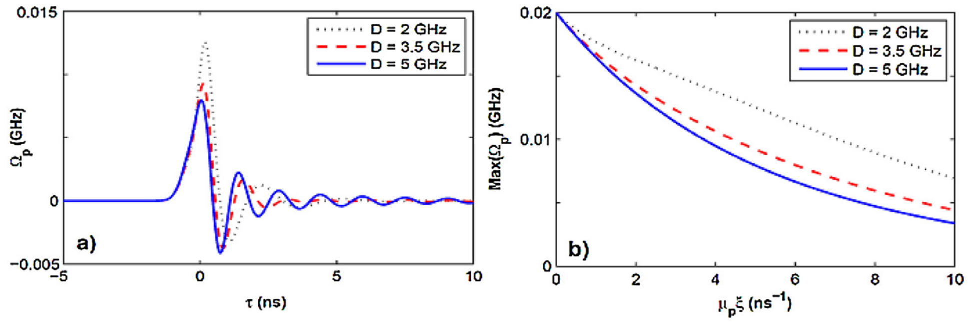

Fig. 3. Temporal evolution of the probe pulse at optical depth $\mu_p\xi = 5\,\text{ns}^{-1}$ (a) and variation of probe peak versus optical depth (b) at different Doppler widths D when $\tau_0 = 1$ ns and $\Omega_{c0} = 10$ GHz.

where, the total Hamiltonian H consists of the free atomic (H_{at}) and interacting (H_{int}) parts. Under the electric dipole- and rotating- wave approximations, the part H_{int} can be given by

$$H_{int}(z,t) = \Omega_p(z,t)e^{-i\omega_p t - ik_p z}|1\rangle\langle 2| + \Omega_c(z,t)e^{-i\omega_c t - ik_c z}|2\rangle\langle 3| + \text{c.c}, \tag{4}$$

with,

$$\Omega_p(z,t) = d_{12}\mathscr{E}_{p0}f_p(z,t)/\hbar \equiv \Omega_{p0}f_p(z,t), \tag{5a}$$

$$\Omega_c(z,t) = d_{23}\mathscr{E}_{c0}f_c(z,t)/\hbar \equiv \Omega_{c0}f_c(z,t), \tag{5b}$$

are the Rabi frequencies those induced by the probe and coupling laser fields, respectively; $\Omega_{p0} = d_{12}\mathscr{E}_{p0}/\hbar$ and $\Omega_{c0} = d_{23}\mathscr{E}_{c0}/\hbar$ are the peak values of the Rabi-frequencies; d_{ij} is the electric dipole matrix element of the transition $|i\rangle \leftrightarrow |j\rangle$.

Under the above approximations, the density matrix Eq. (3) is transformed as

$$\dot\rho_{11} = \Gamma_{21}\rho_{22} + \frac{i}{2}\Omega_p^*\rho_{21} - \frac{i}{2}\Omega_p\rho_{12}, \tag{6a}$$

$$\dot\rho_{22} = -\Gamma_{21}\rho_{22} + \Gamma_{32}\rho_{33} + \frac{i}{2}\Omega_p^*\rho_{12} - \frac{i}{2}\Omega_p\rho_{21} + \frac{i}{2}\Omega_c^*\rho_{32} - \frac{i}{2}\Omega_c\rho_{23}, \tag{6b}$$

$$\dot\rho_{33} = -\Gamma_{32}\rho_{33} - \frac{i}{2}\Omega_c^*\rho_{32} + \frac{i}{2}\Omega_c\rho_{23}, \tag{6c}$$

$$\dot\rho_{12} = -(i\Delta_p + \gamma_{12})\rho_{12} + \frac{i}{2}\Omega_p(\rho_{22} - \rho_{11}) - \frac{i}{2}\Omega_c^*\rho_{13}, \tag{6d}$$

$$\dot\rho_{23} = -(i\Delta_c + \gamma_{23})\rho_{23} + \frac{i}{2}\Omega_c(\rho_{33} - \rho_{22}) + \frac{i}{2}\Omega_p^*\rho_{13}, \tag{6e}$$

$$\dot\rho_{13} = -(i\Delta + \gamma_{13})\rho_{13} + \frac{i}{2}\Omega_p\rho_{23} - \frac{i}{2}\Omega_c\rho_{12}, \tag{6f}$$

where, $\Delta_p = \omega_p - \omega_{21}$ and $\Delta_c = \omega_c - \omega_{32}$ are the frequency detuning of the coupling and probe laser fields, respectively; γ_{ij} is a parameter that represented with the decay rates Γ_{ij} by [34]

$$\gamma_{ij} = \frac{1}{2}\left(\sum_{E_k<E_i}\Gamma_{ik} + \sum_{E_l<E_j}\Gamma_{jl}\right). \tag{7}$$

The density matrix elements ρ_{ik} are restricted by $\rho_{11} + \rho_{22} + \rho_{33} = 1$ and $\rho_{ki} = \rho_{ik}^*$.

Under the slowly varying envelope approximation, each electric field is followed by

$$\left(\frac{\partial}{\partial z} + \frac{1}{c_0}\frac{\partial}{\partial t}\right)E_l(z,t) = -\frac{i\omega_l}{2\varepsilon_0 c_0}P_l(z,t). \tag{8}$$

In Eq. (8), c_0 is the speed of light in vacuum, $P_l(z,t)$ (with $l = p$ or c) is the macroscopic polarization of the medium which is given by

$$P_l(z,t) = Nd_{n2}\rho_{2n}(z,t)e^{i(\omega_l t - k_l z)}, \tag{9}$$

where, N is the density of the atoms, $n = 1$ or 3 for the probe or coupling field, respectively. Substituting Eq. (9) into Eq. (8) we obtain a pair of coupled equations for the coupling and probe Rabi frequencies:

$$\left(\frac{\partial}{\partial z} + \frac{1}{c_0}\frac{\partial}{\partial t}\right)\Omega_p(z,t) = -2i\mu_p\rho_{12}(z,t), \tag{10a}$$

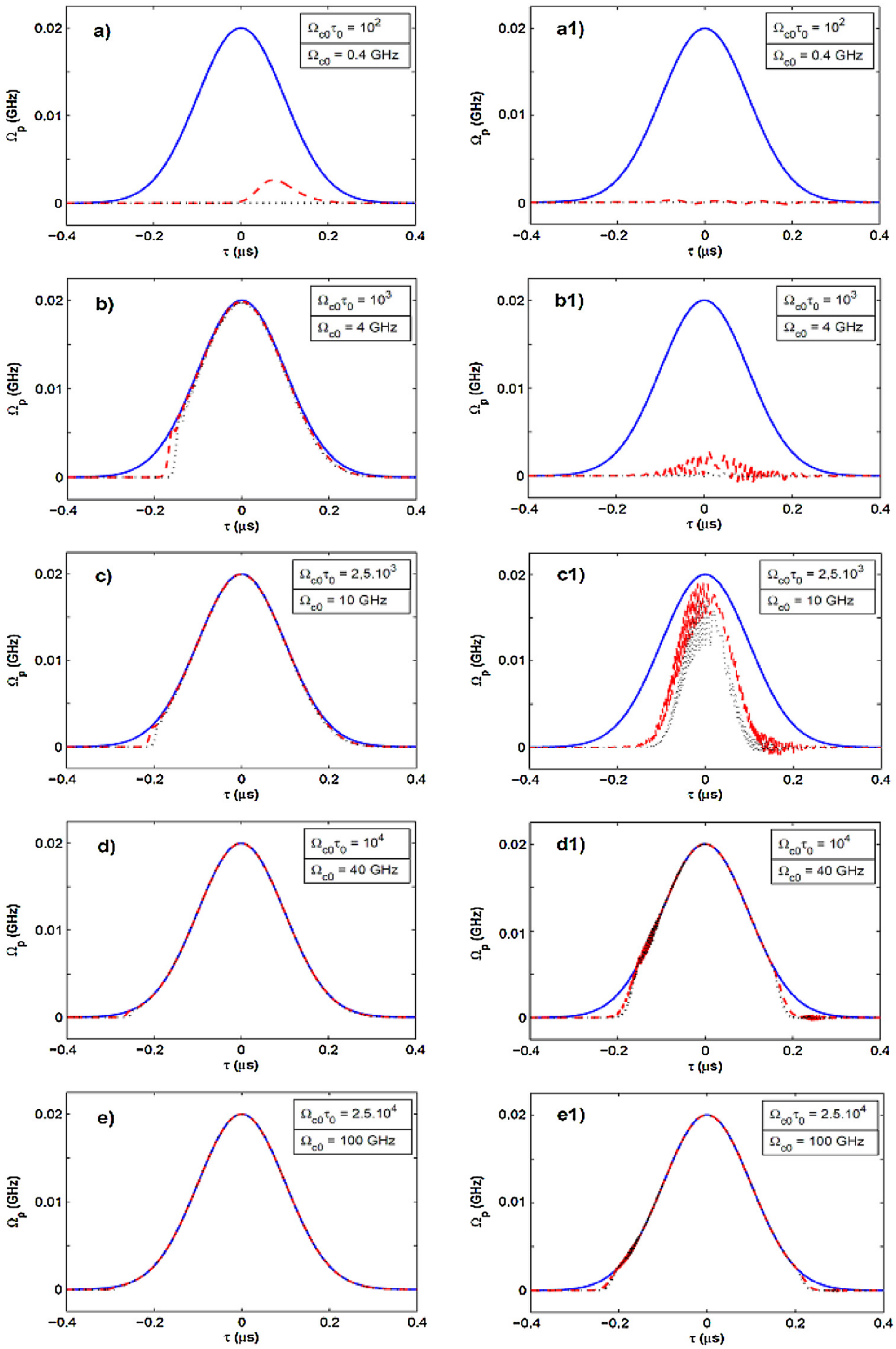

Fig. 4. Temporal evolution of the probe pulse at $\mu_p\xi = 0$ (*solid*), $5\,\text{ns}^{-1}$ (*dashed*), and $10\,\text{ns}^{-1}$ (*dotted*) when $\tau_0 = 0.25\,\mu s$ in the absence (*left*) and presence (*right*) of the Doppler broadening, respectively.

$$\left(\frac{\partial}{\partial z} + \frac{1}{c_0}\frac{\partial}{\partial t}\right)\Omega_c(z,t) = -2i\mu_c\rho_{23}(z,t), \tag{10b}$$

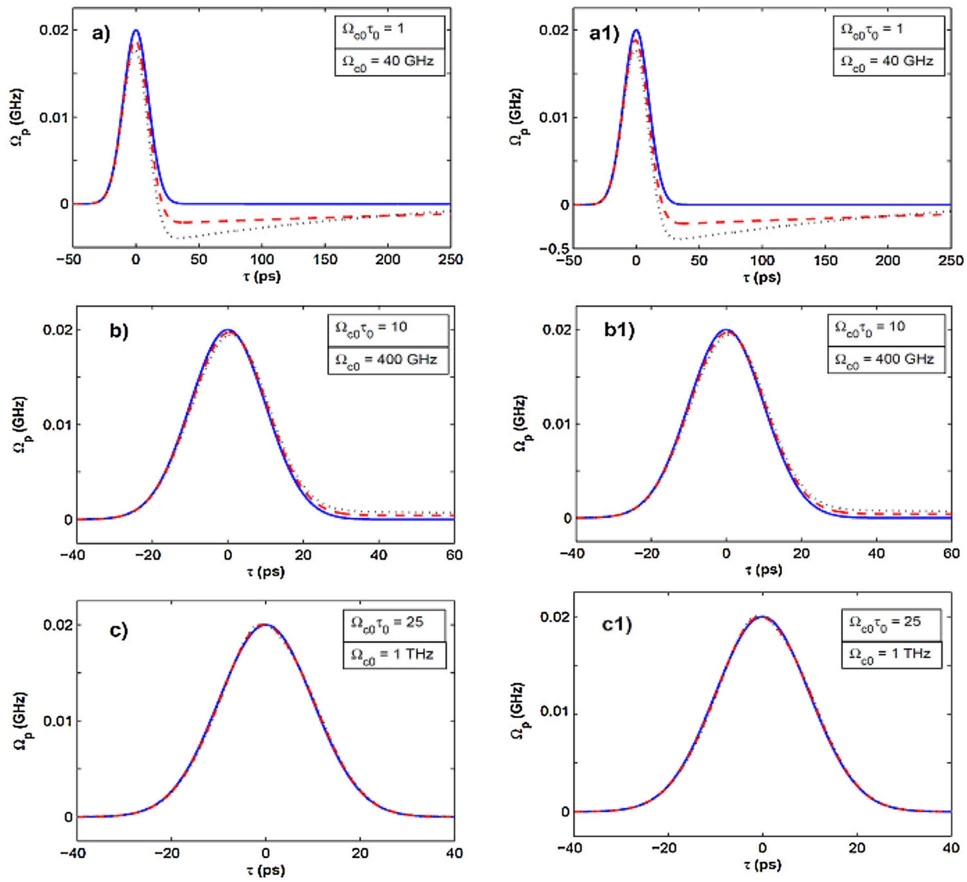

Fig. 5. Temporal evolution of the probe pulse at $\mu_p\xi = 0$ (*solid*), 5 ns^{-1} (*dashed*), and 10 ns^{-1} (*dotted*) when $\tau_0 = 25$ ps in the absence (*left*) and presence (*right*) of the Doppler broadening, respectively.

where,

$$\mu_m = \frac{\omega_m N |d_{n2}|^2}{2\varepsilon_0 c_0 ?}. \tag{11}$$

It is convenient to transform Eq. (6) and Eq. (10) into a local frame with new variables $\xi = z$ and $\tau = t - z/c_0$. In this new frame Eq. (6) will be the same with a substitution $t \to \tau$ and $z \to \xi$, whereas Eq. (10) are rewritten as

$$\frac{\partial}{\partial \xi}\Omega_p(\xi,\tau) = -2i\mu_p\rho_{12}(\xi,\tau), \tag{12a}$$

$$\frac{\partial}{\partial \xi}\Omega_c(\xi,\tau) = -2i\mu_c\rho_{23}(\xi,\tau). \tag{12b}$$

For a presence of Doppler broadening, the frequency detuning of the coupling and probe fields are shifted according as the following relations

$$\Delta_p(v) = \Delta_p^0 - \omega_p v/c_0, \tag{13a}$$

$$\Delta_c(v) = \Delta_c^0 - \omega_c v/c_0, \tag{13b}$$

where, v is the velocity of the moving atom along the z axis, $\Delta_p^0 \equiv \Delta_p(v=0) = \omega_p - \omega_{21}$ and $\Delta_c^0 \equiv \Delta_c(v=0) = \omega_c - \omega_{32}$ are the frequency detuning of the probe and coupling fields for the rest atom ($v = 0$), respectively. Under a thermal equilibrium, the velocity distribution of atoms is represented by the Maxwell-Boltzmann distribution given as

$$g(\Delta) = \frac{1}{\sqrt{\pi} k v_m}\exp\left(-\left(\left(\Delta_p - \Delta_p^0\right)/kv_m\right)^2\right), \tag{14}$$

where, v_m is the most probable velocity that corresponds to a Doppler width D given by

$$D = 2\sqrt{\ln 2}\, v_m\omega_p/c_0. \tag{15}$$

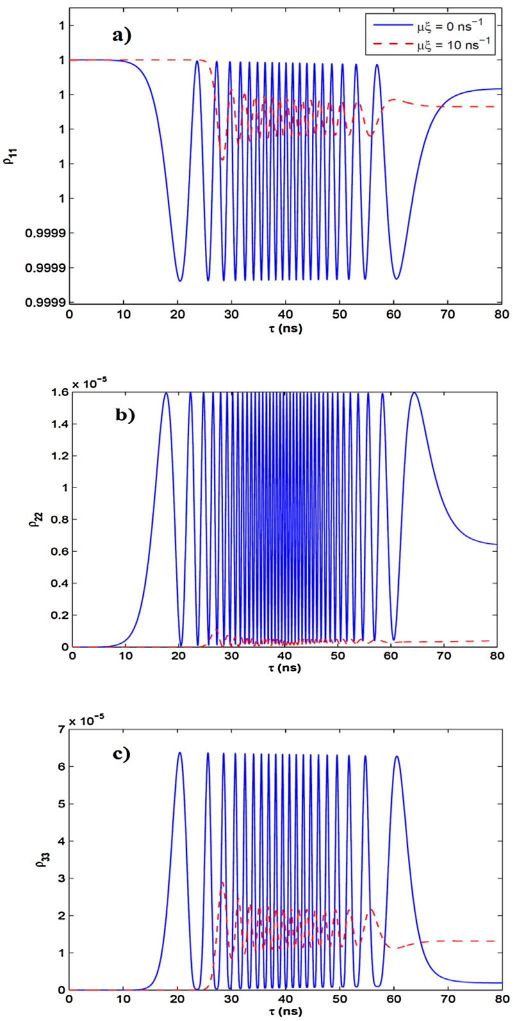

Fig. 6. Variation of the populations at different optical depths $\mu_p\xi = 0$ (*solid line*) and $\mu_p\xi = 10\,\text{ns}^{-1}$ (*dashed line*) in the presence of coupling laser field.

In the presence of Doppler broadening the evolution of the probe and coupling fields in Eq. (12) should be integrated over velocity as

$$\frac{\partial}{\partial\xi}\Omega_p(\xi,\tau) = -2i\mu_p\int_{-\infty}^{\infty} g(\Delta)\rho_{12}(\xi,\tau,\Delta)d\Delta, \tag{16a}$$

$$\frac{\partial}{\partial\xi}\Omega_c(\xi,\tau) = -2i\mu_c\int_{-\infty}^{\infty} g(\Delta)\rho_{23}(\xi,\tau,\Delta)d\Delta. \tag{16b}$$

3. Numerical simulations

In order to study the propagation dynamics of the probe pulse $\Omega_p(\xi,\tau)$, we solve numerically Eq. (6) and Eq. (16) by using the four-order Runge-Kutta and finite difference methods, respectively. The medium is ^{87}Rb atomic vapor in which the states $|1\rangle$, $|2\rangle$, and $|3\rangle$correspond to $5S_{1/2}(F=1)$, $5P_{1/2}(F'=2)$, and $5D_{3/2}(F''=3)$, respectively. The atomic and laser parameters are chosen as (the decay constants and linewidth are given in unit of 2π) [34,35]: $\Gamma_{21} = 6$ MHz, $\Gamma_{32} = 1$ MHz, $N = 10^{15}$ atoms/m^3, $d_{21} = 2.53\times10^{-29}C.m$; $\lambda_p = 795$ nm, $\lambda_c = 762$ nm, $\Omega_{p0} = 0.02$ GHz, and $\Delta_p = \Delta_c = 0$. Both the probe and coupling pulses are chosen with the same duration τ_0.

As a first step of consideration, we study the propagation of the probe laser pulse at a duration $\tau_0 = 25$ ns which is nearly the same to life time of the excited state $|2\rangle$ without Doppler broadening. The temporal evolution of the probe pulse $\Omega_p(\xi,\tau)$ at different values of optical depth $\mu_p\xi$ and coupling pulse area is presented in the left column of Fig. 2.

It is shown in Fig. 2 (*left column*) that for a small value of the coupling pulse area ($\Omega_{c0}\tau_0 \le 25$) the probe pulse is almost collapsed due to resonant absorption of the atomic medium (Fig. 2a). When the pulse area of the coupling laser increases, the front edge of the probe pulse is still distorted but the trailing edge approaches a transparency regime earlier (Figs. 2c and d). As explained by Harris and coworker [15,33], such a distortion of the front edge of the probe pulse is due to the energy loss for preparation of EIT formation. In particular, when the coupling pulse area approaches to a value $\Omega_{c0}\tau_0 = 5 \times 10^3$ (Fig. 2e), the probe pulse is almost unchanged, namely an ideal EIT or soliton is established.

For the presence of Doppler broadening, we study propagation of the probe pulse at a Doppler width $D = 3.15$ GHz which corresponds to the room temperature, as illustrated in the right column of Fig. 2. As we known that the Doppler effect reduces atomic coherence [34], thus it reduces EIT efficiency. Therefore, to reach an EIT regime of probe pulse a larger coupling pulse area is needed (see Figs. 2e and e1). In order to further study the influence of the Doppler broadening we plot temporal profile of the probe pulse (Fig. 3a) and its peak amplitude (Fig. 3b) at different values of the Doppler width D.

From Fig. 3 one can see that when the Doppler width D increases (with a moderate value of coupling pulse area) the peak amplitude of probe pulse decreases (Fig. 3b) whereas the trailing edge of pulse separates into several sub-pulses with stronger oscillating amplitudes (Fig. 3a).

Next, influence of the coupling pulse duration τ_0 on propagation of the probe pulse at a given Doppler width (D = 3.15 GHz) is studied by simulating at longer ($\tau_0 = 0.25\,\mu s$) and shorter ($\tau_0 = 25\,ps$) duration as shown in Figs. 4 and 5, respectively. It can be seen that by increasing the coupling laser intensity one can reach to an EIT regime of the probe pulse in both cases of absence and presence of the Doppler broadening. However, the EIT regime can be easily achieved with a smaller pulse area in a shorter pulse region. This is due to short pulse (much shorter life-time of the excited state) may lead to additional effect (self-induced transparency [26]) whereas the long pulse duration (longer life-time) is strongly absorbed [33]. Nevertheless, to reach the EIT regime in *ps* region of pulse duration, larger coupling laser intensity is needed in comparing with the longer pulse regions. On the other hand, by comparing Figs. 2, 4 and 5 we can see an efficient influence of Doppler broadening in the long duration regions (see Fig. 4e and e1) and it can be ignored in short duration regions (see Fig. 5e and e1).

Finally, we study the variation of populations in a steady state at optical depths, $\mu_p\xi = 0$ (*solid line*) and $\mu_p\xi = 10\,\text{ns}^{-1}$ (*solid line)* when an EIT established that corresponds to the case in Fig. 2b. The population variations are shown in Fig. 6. At $\mu_p\xi = 0$ both light pulses are exactly coherent and the atoms exhibit strong Rabi oscillations (solid lines). When both probe and coupling pulses travel in the medium and the EIT regime established, the population is almost trapped in the state $|1\rangle$ (see Fig. 6a). Therefore, the state $|1\rangle$ does not couple to state $|2\rangle$, *i.e.,* the absorption for the probe pulse is vanished and the oscillation of populations are much less pronounced. Such variations of populations are similar to those obtained in [15] for the case of a three-level lambda-type system.

4. Conclusions

We have studied the propagation of a pair of laser pulses in a three-level cascade medium under Doppler broadening. Influences of the coupling pulse area and Doppler width on EIT formation of probe pulse are studied in a wide region from micro- to pico- second of pulse duration. It has been shown that larger coupling pulse area is needed to establish an EIT regime at longer pulse duration. On the other hand, influence of the Doppler broadening can be ignored in short pulse duration (from *ps*) but it is efficient in the long pulse duration regions (from ns). Furthermore, at moderate values of the coupling pulse area, the Doppler-effect may enhance the amplitude of oscillations at trailing of the probe pulse. In the EIT regime, most populations are trapped in a steady dark state at the ground state and hence the oscillations in populations are much less pronounced. The obtained results can find interesting applications in all optical switching, quantum information processing, and transmission.

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