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The influence of the laser on acoustic phonon amplification in parabolic potential well

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Abstract
Phonon amplification in low-dimensional material structures has primary important applications for engineering instrumentation applications such as phonon spectrometers, phonon lasers, ultrafast optical modulators as well as other devices based on magneto, acoustoelectric and thermal effects. In this paper, we report our theoretical calculation of the acoustic phonon rate in a parabolic semiconductor quantum well. Using the method of quantum kinetic equations, we have found the expression for the acoustic phonon growth rate and the condition for this phonon increase. The analysis results for the gain factor for the acoustic phonons depend on the temperature, amplitude and frequency of the laser field. We numerically calculate the rate of acoustic phonon excitation by the absorption of laser field energy at different temperatures.

Keywords Acoustic phonons · Quantum kinetic equations · Phonon amplification

1 Introduction

The development of methods for generating and amplifying coherent phonons in materials to exploit their use in construction devices such as high-frequency phonon spectrometers (Kharel et al. 2019; Huang and Jing 2019), optical modulators (Sun et al. 2016), phonon lasers (Li et al. 2021; Cui et al. 2021), electrical, magnetic and thermal devices are an important issue in the applied research of low-dimensional materials (Nunes and Fonseca 2012).

The phenomenon of increasing negative phonons and optical phonons, parametric resonances in low-dimensional materials has been studied both theoretically and experimentally in recent times (Zhao et al. 2013; Nunes 2014; Dompreh et al. 2016; Shinokita et al. 2016a; Nafees and Ansari 2265; Nguyen Tien Dung 2021). The main results of these papers are that by absorption of laser field energy, the interaction of the laser field with electron can lead to the excitation of higher harmonics and the amplification of phonon. With the development of modern experimental technology, the fabrications of low- dimensional structures are possible. In reality, phonon amplification by absorption of laser radiation in such confined structures would characterize the electron–phonon interaction.

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The study of quantum transport theories based on the quantum kinetic equation method (QKEM) is a useful tool to investigate the multiphoton absorption process. Using the quantum kinetic equation method, we suggested a trustful acoustic phonon excitation with a linear form. Using method QKEM, we can determine the analytic expression of the phonon growth rate with theoretical models with different low-dimensional materials such as quantum holes, semiconductor superlattices, quantum wires, etc.(Nguyen Tien Dung 2021; Derkacs et al. 2008; Komirenko et al. 2000; Wang et al. 2014; Shinokita et al. 2016b; Nues et al. 2002). From this expression, we can investigate the influence of temperature, laser parameters, material parameters.. on phonon increase rate.

In this paper, we start from the Hamiltonian of the electron–phonon system in a Semiconductor Quantum Well (SQW) with parabolic potential under an intense laser field; we derive a quantum kinetic equation for phonon in SQW in the case of the multiphoton absorption process. Then, we calculate the phonon excitation rate for the two cases of the electron gas that are on-degenerative and degenerative. Finally, we numerically calculate the acoustic phonon excitation rate (APER) in a specific SQW with parabolic potential to illustrate the mechanism of the phonon amplification.

2 Quantum kinetic equation for phonon in a SQW

The research model is a semiconductor quantum well with parabolic potential made from semiconductors with nearly identical lattice structures. A simple model for a SQW with parabolic potential $V(z) = \frac{kz^2}{2}$, where k is the force constant of the oscillator. We assume the electromagnetic wave of the laser field propagates in a direction perpendicular to the contact layers and penetrates deep into the sample. The planar polarized electromagnetic waves have electric field strength vectors: $\vec{E} = \vec{e}_\perp E_0 \sin \Omega t$ ($\vec{e}_\perp$ is the unit vector parallel to the pit well). Assume a non-decreasing potential for phonons (three-dimensional phonons). $\vec{A}$ is the potential vector, depending on the external field:

$$\vec{A} = \vec{A}_0 \cos \Omega t,\ A_0 = cE_0/\Omega \tag{1}$$

The energy of the electron in the potential pit is quantized in a direction parallel to the surface normal. In which a two-dimensional electron gas is confined by SQW potential along the z direction and electrons are free on the x–y plane, the wave vector is $\vec{k}_\perp$. It is well known that its energy spectrum is quantized into discrete levels in the z direction. $\varepsilon_n(\vec{k}_\perp)$ is the energy spectrum of the electron for the wave $\vec{k}_\perp$, it takes the form (Bhattacharya et al., 2011):

$$\varepsilon_n(\vec{k}_\perp) = \hbar\omega\left(n + \frac{1}{2}\right) + \frac{\hbar^2\vec{k}_\perp^2}{2m_e} = \varepsilon_n + \frac{\hbar^2\vec{k}_\perp^2}{2m_e};\quad \text{with}\, n = 0, 1, 2 \ldots \tag{2}$$

e and m_e are the charge and the effective mass of the electron, $\omega = \sqrt{k/m_e}$.

The Hamiltonian for the system of the electrons and phonons in the case of the presence of the laser field is written as (Nguyen Tien Dung 2021; Derkacs et al. 2008; Komirenko et al. 2000; Wang et al. 2014; Shinokita et al. 2016b; Nues et al. 2002):

$$\hat{H}(t) = \hat{H}_e + \hat{H}_{ph} + \hat{H}_{e-ph} = \sum_{\vec{k}_\perp, n_1} \varepsilon_{n_1}\left(\vec{k}_\perp - \frac{e}{c\hbar}\vec{A}(t)\right) a^{(n_1)^+}_{\vec{k}_\perp} a^{(n_1)}_{\vec{k}_\perp} + \sum_q \hbar\omega_{\vec{q}}\, b^+_{\vec{q}}\, b_{\vec{q}} + \sum_{\vec{k}_\perp, \vec{q}, n_1, n_2} C_{n_1,n_2}(\vec{q})\, a^{(n_2)^+}_{\vec{k}_\perp+\vec{q}}\, a^{(n_1)}_{\vec{k}_\perp}(b_{\vec{q}} + b^+_{-\vec{q}}) \tag{3}$$

where $a^{(n)^+}_{\vec{k}_\perp}$ and $a^{(n)}_{\vec{k}_\perp}$ are the creation and annihilation operators of electron in the n state, $b^+_{\vec{q}}$ and $b_{\vec{q}}$ are the creation and annihilation operators of phonon, $\varepsilon_{\vec{q}} = \hbar\omega_{\vec{q}}$ is phonon energy for wave vector $\vec{q}$.

$$C_{n_1 n_2}(\vec{q}) = C_{\vec{q}} I_{n_1 n_2}(\vec{q}) \tag{4}$$

where $C_{\vec{q}}$ is the electron–phonon interaction constant and $I_{n_1 n_2}(\vec{q})$:

$$I_{n_1 n_2}(q_z) = \int_{-\infty}^{+\infty} H_{n_1}(z)\, H_{n_2}(z) \exp(iq_z z) dz \tag{5}$$

with $H_n(z) = \frac{1}{2^n!}\sqrt{\frac{\alpha}{\pi}}(-1)^n e^{\frac{\alpha z^2}{2}} \frac{d^n}{dz^n} e^{-\alpha z^2}$ is wave function.

Similar to [2, 42], the average number of phonons in the quantum well is determined by the expression:

$$N_{\vec{q}}(t) = \left\langle b^+_{\vec{q}} b_{\vec{q}} \right\rangle_t = Tr\left\{ \hat{\rho} b^+_{\vec{q}} b_{\vec{q}} \right\}_t$$

where $N_{\vec{q}}(t) =< b^+_{\vec{q}} b_{\vec{q}} >_t$, the symbol $< X >_t$ means the usual thermodynamic average of operator X.

The quantum kinetic equation for $N_{\vec{q}}(t)$ has form:

$$\frac{\partial N_{\vec{q}}(t)}{\partial t} = i\hbar \frac{\partial}{\partial t}\left\langle b^+_{\vec{q}} b_{\vec{q}} \right\rangle_t = \left\langle b^+_{\vec{q}} b_{\vec{q}}, \hat{H}(t) \right\rangle_t \tag{6}$$

Substituting (3) into (6), we perform calculations that lead to the differential equation:

$$\begin{aligned}
\frac{\partial N_{\vec{q}}(t)}{\partial t} =& \frac{1}{\hbar^2} \sum_{\vec{k}, n_1, n_2} \left| C_{n_1 n_2}(\vec{q}) \right|^2 \sum_{\ell, \sigma=-\infty}^{+\infty} J_\sigma(\Lambda/\hbar\Omega) J_\ell(\Lambda/\hbar\Omega) exp[i(\ell - \sigma)\Omega t] \\
&\times \int_{-\infty}^{t} \left\{ \left[\left(N_{\vec{q}}(t) + 1\right) f_{n_2}(\vec{k}_\perp + \vec{q}) \left(1 - f_{n_1}(\vec{k}_\perp)\right) - N_{\vec{q}}(t) f_{n_1}(\vec{k}_\perp) \left(1 - f_{n_2}(\vec{k}_\perp + \vec{q})\right) \right] \right. \\
&\times \exp\left[\frac{i}{\hbar}\left(\varepsilon_{n_2}(\vec{k}_\perp) - \varepsilon_{n_1}(\vec{k}_\perp - \vec{q}) - \varepsilon_{\vec{q}} - \ell\hbar\Omega \right)(t - t') \right] \\
&+ \int_{-\infty}^{t} \left\{ \left[\left(N_{\vec{q}}(t) + 1\right) f_{n_2}(\vec{k}_\perp) \left(1 - f_{n_1}(\vec{k}_\perp - \vec{q})\right) - N_{\vec{q}}(t) f_{n_1}(\vec{k}_\perp) \left(1 - f_{n_2}(\vec{k}_\perp)\right) \right] \right. \\
&\times \exp\left[\frac{i}{\hbar}\left(\varepsilon_{n_2}(\vec{k}_\perp) - \varepsilon_{n_1}(\vec{k}_\perp - \vec{q}) - \varepsilon_{\vec{q}} - \ell\hbar\Omega \right)(t - t') \right] \Big\} dt'
\end{aligned} \tag{7}$$

where $J_\ell(z)$ is Bessel function, $f_n(\vec{k}_\perp)$ is the distribution function of the electron, $\Lambda = e\hbar\vec{E}_0\vec{q}_\perp/(m_e\Omega)$.

Equation (7) has the same form as the kinetic equations for phonon of F. Peng (Peng and Nan-xian 1992) and P. Zhao (Zhao 1994) which these authors established by other methods. The advantage of the method we have used here is due to the second quantization method. Using this method, some quantum properties of a homogeneous particle system will be expressed in terms of wave and energy vectors.

It can be said that solving Eq. (7) in the general case is impossible. However, in some approximate cases considering electron–phonon interaction as a disorder, and the number of phonons satisfies the condition $N_{\vec{q}}(t) \gg 1$, then Eq. (7) has the following simple form:

$$\begin{aligned}\frac{\partial N_{\vec{q}}(t)}{\partial t} &= \frac{1}{\hbar^2}\sum_{\vec{k},n_1,n_2}\left|C_{n_1n_2}(\vec{q})\right|^2\sum_{\ell=-\infty}^{+\infty}J_\ell^2(\Lambda/\hbar\Omega)\int_{-\infty}^{t}dt'\,N_{\vec{q}}(t')\\ &\times\left\{\left[f_{n_2}(\vec{k}_\perp+\vec{q})-f_{n_1}(\vec{k}_\perp)\right]\exp\left[\frac{i}{\hbar}\left(\varepsilon_{n_2}(\vec{k}_\perp+\vec{q})-\varepsilon_{n_1}(\vec{k}_\perp)-\varepsilon_{\vec{q}}-\ell\hbar\Omega\right)(t-t')\right]\right.\\ &\left.+\left[f_{n_1}(\vec{k}_\perp)-f_{n_2}(\vec{k}_\perp-\vec{q})\right]\exp\left[-\frac{i}{\hbar}\left(\varepsilon_{n_1}(\vec{k}_\perp)-\varepsilon_{n_2}(\vec{k}_\perp-\vec{q})-\varepsilon_{\vec{q}}-\ell\hbar\Omega\right)(t-t')\right]\right\}\end{aligned} \tag{8}$$

this equation will be used to determine the phonon growth rate in the semiconductor quantum well.

3 Phonon excitation rate in a SQW

These results allow (Nunes 2014) one to introduce the kinetic equation for phonon number of the q mode:

$$\frac{\partial N_{\vec{q}}(t)}{\partial t} = \gamma_{\vec{q}}N_{\vec{q}}(t) \tag{9}$$

where $\gamma_{\vec{q}}$ are parameters that determine the evolution of the phonon number $N_{\vec{q}}(t)$ in time due to the interaction with the electrons. If $\gamma_{\vec{q}} > 0$ the phonon population grows with time, whereas for $\gamma_{\vec{q}} < 0$ we have damping.

Referring to Eq. (8) phonon excitation rate in (9) is:

$$\begin{aligned}\gamma_{\vec{q}} =& \frac{\pi}{\hbar}\sum_{\vec{k},n_1,n_2}\left|C_{n_1n_2}(\vec{q})\right|^2\left[f_{n_2}(\vec{k}+\vec{q})-f_{n_1}(\vec{k})\right]\left\{\delta\left(\varepsilon_{n_2}(\vec{k}+\vec{q})-\varepsilon_{n_1}(\vec{k})-\varepsilon_{\vec{q}}-\Lambda\right)\right.\\ &\left.+\delta\left(\varepsilon_{n_2}(\vec{k}+\vec{q})-\varepsilon_{n_1}(\vec{k})-\varepsilon_{\vec{q}}+\Lambda\right)\right\}\end{aligned} \tag{10}$$

In the strong-field limit, $\Lambda >> \hbar\Omega$ and the argument of the Bessel function in Eq. (5) is larger. For large values of argument, the Bessel function is small except when the order is equal to the argument. The sum over ℓ in Eq. (10) may then be written approximately:

$$\sum_{\ell=-\infty}^{\infty}J_\ell^2\left(\frac{\Lambda}{\hbar\Omega}\right)\delta(E-\ell\hbar\Omega) = \frac{1}{2}[\delta(E+\Lambda)+\delta(E-\Lambda)] \tag{11}$$

Here $E = \varepsilon_n(\vec{k}+\vec{q}) - \varepsilon_n(\vec{k}) - \varepsilon_{\vec{q}}$. The first delta function corresponds to the absorption and the second one corresponds to the emission of $\Lambda/(\hbar\Omega)$ photons. In other words, in the strong-field limit only multiphoton processes is significant and the electron–phonon collision takes place with the emission and absorption of $\Lambda/(\hbar\Omega)$ photons. Substituting Eq. (11) into Eq. (10), the phonon excitation rate becomes $\gamma_{\vec{q}} = \gamma_{\vec{q}}^{+} + \gamma_{\vec{q}}^{-}$, where:

$$\gamma_{\vec{q}}^{(\pm)} = \frac{\pi}{\hbar}\sum_{\vec{k},n_1,n_2}\left|C_{n_1n_2}(\vec{q})\right|^2\left[f_{n_2}(\vec{k}+\vec{q}) - f_{n_1}(\vec{k})\right]\,\delta\left(\varepsilon_{n_2}(\vec{k}+\vec{q}) - \varepsilon_{n_1}(\vec{k}) - \varepsilon_{\vec{q}} \pm \Lambda\right) \tag{12}$$

In the following, we will calculate for the case in which the electron gas is non-degenerative. In this case, we may simplify the carrier distribution function by using the Boltzmann distribution function $f_n(\vec{k}_\perp) = \exp\left[\beta\left(\varepsilon_F - \varepsilon_n(\vec{k}_\perp)\right)\right]$. From Eq. (12), for the case in which $\vec{q} = \vec{q}_z + \vec{q}_\perp$, we obtain the expression for the rate of phonon excitations:

$$\gamma_{\vec{q}}^{(\pm)} = \frac{S}{\hbar^4 q_\perp^2}\left(\frac{m_e^3}{8\pi^3\beta}\right)^{1/2}\sum_{n_1,n_2}\left|C_{n_1n_2}(\vec{q})\right|^2\exp\left[\beta\left(\varepsilon_F - \hbar\omega\left(n_1+\frac{1}{2}\right)\right)\right]$$
$$\times\exp\left[-\frac{\beta m_e}{2\hbar^2 q_\perp^2}\left(\frac{\hbar^2 q_\perp^2}{2m_e} + \hbar\omega(n_2-n_1) - \hbar\omega_{\vec{q}} \mp \Lambda\right)^2\right]\left\{\exp\left[-\beta\left(\hbar\omega_{\vec{q}} \mp \Lambda\right)\right] - 1\right\} \tag{13}$$

The upper and lower + (-) sign in the Eq. (13) corresponds to the absorption (emission) of a photon with energy $\hbar\Omega$ of the laser field. Here $\beta = 1/\left(k_B T\right)$, k_B is the Boltzmann constant and T is the temperature of the system.

Next we discuss the condition to obtain expression (13) and the condition to have the phonon increase according to the theoretical calculation:

a) The result is obtained when the argument of the Delta-Dirac function in (11) is zero:

is the condition about the quality to have the phonon increased effects.

$$\frac{\hbar k_\perp}{m_e} = \frac{\hbar q_\perp}{2} + \frac{m_e}{\hbar q_\perp}\left(\varepsilon_{n_2} - \varepsilon_{n_1} + \Lambda - \hbar\omega_{\vec{q}}\right) > \frac{\hbar q_\perp}{2} \to 2k_\perp > q_\perp \tag{14}$$

b) From Eq. (13), we see that the emission of many photons (lower sign) always gives $\gamma_{\vec{q}}^{(\pm)} < 0$. The process of absorbing many photons (upper sign) can lead to an increase in phonon $\gamma_{\vec{q}}^{(\pm)} > 0$. Analyzing Eq. (13) we can obtain the conditions for the phonon amplification. From the condition $\gamma_{\vec{q}}^{(\pm)} > 0$, we obtain $\exp\left[-\beta\left(\hbar\omega_{\vec{q}} - \Lambda\right)\right] - 1 > 0$. The condition which the laser field must satisfy is:

$$\Lambda = \frac{\hbar e\vec{q}_\perp\vec{E}_0}{m_e\Omega} > \hbar\omega_{\vec{q}} \tag{15}$$

The condition (1) simply means that if drift velocity of electron $\vec{q}_\perp\vec{E}_0/m_e\Omega$ under the intense laser field, excesses the phonon phase-velocity, a deformation potential for multiphonon excitation can be generated in the SQW.

We calculate the rate of acoustic phonon excitation with $\left|C_{\vec{q}}\right|^2 = \hbar q \xi^2/(\rho v_a V)$. Here, V, ρ, v_a, and ξ are the volume, the density, the acoustic velocity, and the deformation potential constant, respectively. We have the rate of phonon excitations.

For the rate of acoustic phonon excitations:

$$\gamma_{\vec{q}}^{(+)a} = \frac{\xi^2}{2\hbar^3 q_\perp L \rho v_a}\left(\frac{m_e^3}{8\pi^3\beta}\right)^{1/2}\sum_{n_1,n_2}\left|I_{n_1n_2}(\vec{q})\right|^2 \exp\left[\beta\left(\varepsilon_F - \hbar\omega\left(n_1+\frac{1}{2}\right)\right)\right] \times \exp\left[-\frac{\beta m_e}{2\hbar^2 q_\perp^2}\left(\frac{\hbar^2 q_\perp^2}{2m_e} + \hbar\omega\left(n_2-n_1\right) - \hbar\omega_{\vec{q}} - \Lambda\right)^2\right]\left\{\exp\left[-\beta\left(\hbar\omega_{\vec{q}} - \Lambda\right)\right] - 1\right\} \tag{16}$$

In the following, we will calculate for the case in which the electron gas is degenerative. In this case, we may simplify the carrier distribution function by using the Boltzmann distribution function:

$$f_n\left(\vec{k}_\perp\right) = \theta\left(\varepsilon_F - \varepsilon_n\left(\vec{k}_\perp\right)\right) = \begin{cases} 1 \; if \; \varepsilon_F > \varepsilon_n\left(\vec{k}_\perp\right) \\ 0 \text{ if } \varepsilon_F < \varepsilon_n\left(\vec{k}_\perp\right) \end{cases} \tag{17}$$

For the rate of acoustic phonon excitations:

$$\gamma_{\vec{q}}^{(+)a} = \frac{\xi^2 q^2}{\hbar^3 \rho v_a q_\perp L}\sqrt{\frac{m_e^3}{2}}\sum_{n_1,n_2}\left|I_{n_1,n_2}(\vec{q})\right|^2\left\{\left[\left(\varepsilon_F - \varepsilon_{n_1} - \hbar\omega_{\vec{q}} + \Lambda\right) - \frac{m_e}{2\hbar^2 q_\perp^2}\left(\frac{\hbar^2 q_\perp^2}{2m_e} + \hbar\omega\left(n_2-n_1\right) - \hbar\omega_{\vec{q}} + \Lambda\right)^2\right]^{1/2} - \left[\left(\varepsilon_F - \varepsilon_{n_1}\right) - \frac{m_e}{2\hbar^2 q_\perp^2}\left(\frac{\hbar^2 q_\perp^2}{2m_e} + \hbar\omega\left(n_2-n_1\right) - \hbar\omega_{\vec{q}} + \Lambda\right)^2\right]^{1/2}\right\} \tag{18}$$

Therefore, if $\Lambda >> \varepsilon_F$ then the photon emission process can be neglected compared to the absorption process (Peng and Nan-xian 1992; Zhao 1994; Tronconi and Nunces 1986; Nues et al. 1984). The general energy condition for an electron to move from a bonded state to a higher energy state is:

$$\Lambda > max\left(\varepsilon_F, \hbar\omega_{\vec{q}}\right) \tag{19}$$

Note that $\Lambda = e\hbar\vec{E}_0\vec{q}_\perp/\left(m_e\Omega\right)$, for simplicity we assume $\vec{q}$ is directed in the direction of $\vec{E}$, so $\Lambda = e\hbar E_0 q/\left(m_e\Omega\right) = \hbar q v$, where $v = eE_0/(m_e\Omega)$ is the drag velocity of the electron under the action of the laser field. $v_{ph} = \omega_{\vec{q}}/q$ is the phase velocity of the phonon. Thus, the condition $\Lambda > \hbar\omega_{\vec{q}}$ means that the drag velocity v of the electron exceeds the phase velocity of the phonon, and $\Lambda > \varepsilon_F$ means that the drag velocity of the electron exceeds $\varepsilon_F/(\hbar q)$.

For $\gamma_{\vec{q}}^{(+)a}$ to determine then the condition

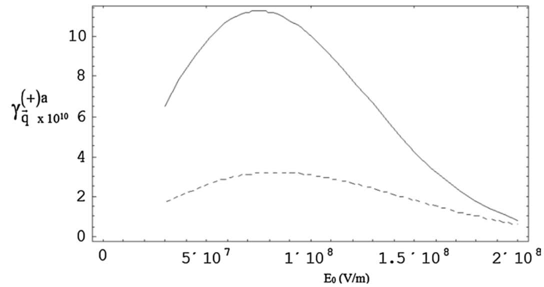

Fig. 1 Dependence of phonon rate increase on laser field amplitude E_0 with frequency $\Omega = 10^{15}$ rad/s, wave number $q = 2 \times 10^8\ m^{-1}$, here T = 100 K solid curve and T = 150 K broken curve

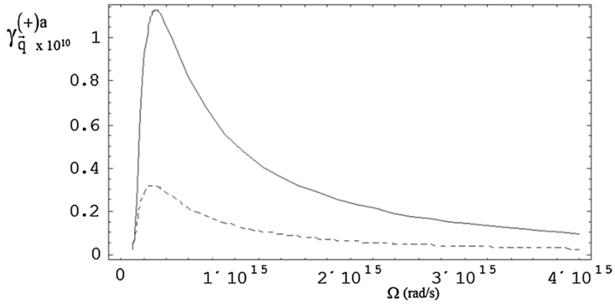

Fig. 2 Dependence of phonon rate increase on laser field frequency Ω with laser field amplitude $E_0 = 2.3 \times 10^{7}$ V/m, wave number $q = 2 \times 10^8\ m^{-1}$, here T = 100 K solid curve and T = 150 K broken curve

$$\begin{cases} \left(\varepsilon_F - \varepsilon_{n_1} - \hbar\omega_{\vec{q}} + \Lambda\right) - \dfrac{m_e}{2\hbar^2 q_\perp^2}\left(\dfrac{\hbar^2 q_\perp^2}{2m_e} + \hbar\omega\left(n_2 - n_1\right) - \hbar\omega_{\vec{q}} + \Lambda\right)^2 \geq 0 \\ \left(\varepsilon_F - \varepsilon_{n_1}\right) - \dfrac{m_e}{2\hbar^2 q_\perp^2}\left(\dfrac{\hbar^2 q_\perp^2}{2m_e} + \hbar\omega\left(n_2 - n_1\right) - \hbar\omega_{\vec{q}} + \Lambda\right)^2 \geq 0 \end{cases} \tag{19}$$

notice (19) we get the condition which the laser field must satisfy is

$$\varepsilon_F \geq \varepsilon_{n_1} + \frac{m_e}{2\hbar^2 q_\perp^2}\left(\frac{\hbar^2 q_\perp^2}{2m_e} + \hbar\omega\left(n_2 - n_1\right) - \hbar\omega_{\vec{q}} + \Lambda\right)^2 \tag{20}$$

this means the amplitude of the external laser field is higher than some threshold amplitude as the condition of the Cerenkov.

4 Numerical results and conclusions

In order to clarify mechanism for the phonon amplification, we numerically calculate the rate of acoustic phonon excitation by the absorption of laser field energy. The parameters used in the calculation are as follows $\hbar = 1.05 \times 10^{-34}\ Js$, $m_e = 0.066 m_0$, with m_0 being the mass of free electron, $e_f = 0.05$ eV, $\rho = 5.32 \times 10^3\ kg/m^3$, $v_a = 5370$ m/s, $\xi = 13.51$ eV, $n_1 = 1$, $n_2 = 2$ for GaAs/$Ga_{1\text{-}x}As_xAl$ (Shinokita et al. 2016a).

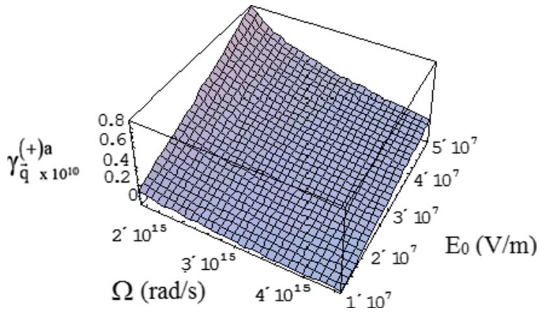

Fig. 3 Dependence of phonon rate increase on laser field frequency Ω and laser field amplitude, at wave number $q = 2 \times 10^8$ m^{-1}

The dependence of the phonon rate increase in the case of non degenerate electron gas on the laser field amplitude with frequency value $\Omega = 10^{15}$ rad/s at two temperatures T = 100 K and T = 150 K is described as Fig. 1.

The dependence of the rate of phonon increase in the case of non degenerate electron gas on Ω laser frequency with the laser field amplitude $E_0 = 2.3 \times 10^7$ V/m at two temperatures T = 100 K and T = 150 K is described in Fig. 2.

Figures 1 and 2 show the existence of field strength and frequency values for maximum phonon growth rates. Comparing the graphs in Figs. 1 and 2 show that when the frequency increases, the rate of increase of phonon increases faster to the maximum value then decreases. The higher the temperature, the lower the maximum value meaning that the number of negative phonons produced decreases. In our opinion, the reason for this is that the higher the temperature, the greater the phonon energy, while the field energy is constant, so the rate of acoustic phonon production decreases. The sensitivity of the rate of phonon production to temperature is a characteristic of negative phonons.

The dependence of the rate of phonon increase in the case of degenerate electron gas on the laser frequency Ω and the laser field amplitude E_0 as shown in Fig. 3.

From Fig. 3, we see that as the frequency and amplitude of the laser field increase, in the case of degenerate electron gas, the rate of increase of phonon increases.

In conclusion, we have analytically investigated the possibility of phonon amplification by absorption of laser field energy in a SQW with parabolic potential well in the case of multiphoton absorption process with non-degenerative and degenerative electron system. Starting from bulk phonon assumption and Hamiltonian of the electron–phonon system in laser field we have derived a quantum kinetic equation for phonon in SQW. However, an analytical solution to the equation can only be obtained within some limitations. Using these limitations for simplicity, we have obtained expressions of the rate of acoustic phonon excitation in the case of multiphoton absorption process. Finally, the expressions are numerically calculated and plotted for a SQW to show the mechanism of the phonon amplification. Similar to the mechanism pointed out by several authors for different models, phonon amplification in a SQW can occur under the conditions that the amplitude of the external laser field is higher than some threshold amplitude as the condition of the Cerenkov.

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