Optical and Quantum Electronics (2022) 54:42
https://doi.org/10.1007/s11082-021-03416-7

Broadband laser-driven creation of entangled state for a nonlinear coupling coupler pumped in one mode

A. Nguyen Tuan[1] · O. Luong Thi Tu[2,3] · L. Chu Van[3] · D. Nguyen Thi[4] · D. Hoang Minh[1] · K. Doan Quoc[5]

Received: 17 August 2021 / Accepted: 16 October 2021

Abstract
We study a system to which two nonlinear oscillators nonlinearly joined together. Either one of the oscillators is pumped by a classical field that is supposed to be separated into two elements, namely a deterministic component and white noise. We point out that the progress of our system is also able to created maximally entangled states which can change dramatically in comparison to the case of parameter concerned to the troublous element of the external field is not present. The reasonable results are affirmed by the comparison to that of previous papers.

Keywords Nonlinear coupler · Bell-like state · Stochastic process · Parameter related to the noisy part · White noise

1 Introduction

Although quantum theory has made great contributions to changing human civilization, many basic problems in its foundation remain unsolved. For a long time, these problems have been the main challenge even for the founders of quantum theory. At the beginning, they were usually expressed by the so-called "gedanken experiments", and treated as the philosophical problems rather than physical ones. However, based on experimental methods developed rapidly in quantum optics in the last time, one can be able to control single quantum systems such as atoms, electrons, ions or photons. Then the gedanken experiments became realizable in practice for better understanding of the foundations of quantum theory.

✉ K. Doan Quoc
doanquockhoa@tdmu.edu.vn

1 Ho Chi Minh City University of Food Industry, Ho Chi Minh City, Vietnam

2 Nghe An College of Education, Vinh, Vietnam

3 Vinh University, 182 Le Duan, Vinh, Vietnam

4 Hong Duc University, 565 Quang Trung, Thanh Hoa, Vietnam

5 Institute of Applied Technology, Thu Dau Mot University, Binh Duong Province, Vietnam

Published online: 07 December 2021

There are two problems, both of which are of special interests. The first relates to measurement. When we obtain a particular feature of the system, we cannot affirm that the system has this feature before measuring it and this feature does not depend on the measurement. Thus, the question that the system has or does not have a particular feature in reality on which does not depend the measurement, is not meaningful. The second problem relates to the nature of quantum coherence between the subsystems within a whole system, namely measurements performed on one subsystem can impact on measurements in other subsystem immediately even if two subsystems are placed very far from each other i.e., quantum theory is not local. These two problems are in contrast with intuitive sense, so they are called Schrödinger's cat paradox and Einstein–Podolsky–Rosen (EPR) paradox (Einstein et al. 1935). These paradoxes are the starting point leading to so-called entanglement states being a basic resource for quantum computing. Scientists have been concerned about the ability to create and control the nonclassical states, which are indeed important to the application for the models of quantum information theory and communication. The research in these fields performed intensively in last twenty years leads to important results which both consolidate the fundamentals of quantum theory and implement for quantum engineering. In practice, the particular interests are the systems can be closed in a finite-dimensional state. Physical system characterized by the Kerr nonlinearities can be generated such quantum states as linear (Pegg et al. 1998; Barnett and Pegg 1999; Ozdemir et al. 2001) or nonlinear (Leoński and Tanaś 1994; Miranowicz et al. 1996; Leoński 1997; Leoński et al. 1997; Leoński and Kowalewska-Kudłaszyk 2011; Leoński and Miranowicz 2001, 2004) quantum scissors. The models of Kerr nonlinear coupler including two nonlinear oscillators which are linear bond at one another and affiliate to one or two externals classical fields have been deliberated (Leoński and Miranowicz 2004; Miranowicz and Leoński 2006; Kowalewska-Kudłaszyk et al. 2014). The model of nonlinear coupler has been grown in instance of the nonlinear (Kowalewska-Kudłaszyk and Leoński 2006; Le Duc and Cao Long 2016) or parametric down-converter coupling coupler (Kowalewska-Kudłaszyk et al. 2012; Kowalewska-Kudłaszyk and Leoński 2014) and linear interaction or parametric process for the external classical fields. The model of nonlinear coupler with three oscillators driven by laser field has been discussed (Kalaga et al. 2016). The set of states portrayed the systems of three qubits for the case of quantum steering was considered (Kalaga and Leoński 2017). Three-qubit interaction model, like entanglement, and coherence were analysed (Kalaga et al. 2018). The second creation of harmonic with the fluctuation of quantum phase in the system of far-off resonance was discussed (Peřinová et al. 2018).

In all above-mentioned models, the external fields are completely monochrome. On the other hand, a real external field is not ever impeccably monochrome. This field often modelled by Gaussian process. In addition, it is difficult to derive exactly analytical mean of the equations of random differentiation for Gaussian process. Almost only the special case of white noise has been well considered and we can reach some interesting results (Cao Long and Trippenbach 1986; Doan Quoc et al. 2012a, b, 2016, 2018). In particular, the Kerr nonlinear coupler that involves two nonlinear oscillators linearly joined together and interact with one or two external fields that modelled by white noises are discussed in Doan Quoc et al. (2016), Doan Quoc et al. (2020); Luong Thi Tu et al. (2020). In our paper, we should expand the coupler involving two nonlinear oscillators nonlinearly joined at one another and this nonlinear coupler is stimulated by a more realistic field in the case width of the external field is taken into account. This means that the external field assumption is decomposed into two elements: white noise and a deterministic constituent. What is mainly interesting from our opinion is that these systems can change the creation of the maximally entangled and finite-dimensional states.

Our work is structured the following: The model of a nonlinear coupler is depicted in Sect. 2. In the next section, calculated results are debated. Finally, the conclusions are inferred.

2 The model of nonlinear coupling coupler

The coupler model studied here is constructed by two nonlinear oscillators, which correspond to the two modes of the field a and b, and coupled to each other by a nonlinear interaction. Furthermore, either one of the oscillators corresponding to mode a linearly connects with external field (Fig. 1). Hence, the effective Hamilton operator to describe this system in picture of the interaction can have the form as

$$\hat{H} = \frac{\chi_a}{2}\left(\hat{a}^+\right)^2\hat{a}^2 + \frac{\chi_b}{2}\left(\hat{b}^+\right)^2\hat{b}^2 + \mu\left(\hat{a}^+\right)^2\left(\hat{b}\right)^2 + \mu^*\left(\hat{b}^+\right)^2(\hat{a})^2 + \sigma\hat{a}^+ + \sigma^*\hat{a}, \tag{1}$$

where $\hat{a}\left(\hat{b}\right)\left(\hat{a}^+\left(\hat{b}^+\right)\right)$ is bosonic annihilation (creation) operator that corresponds to the mode a (b) of the oscillator; the parameters χ_a and χ_b are Kerr nonlinearities of the oscillators a and b, respectively; μ and σ are the coupling strength between two oscillators and the strength of external coherent field, respectively.

We will depict evolution of the system, which is without processes of the damping, in the form the time-dependent wave function as

$$|\psi(t)\rangle = \sum_{p,q=0}^{\infty} c_{pq}(t)|p\rangle_a|q\rangle_b, \tag{2}$$

$c_{pq}(t)$ are amplitudes of the probability for discovering the system in the p-photon and the q-photon states for modes a and b, respectively.

Because our model includes the external pumping by the classical field that is assumed to be separated into two elements, namely a deterministic component and white noise, the total energy of the system examined is not conserved. Thus, one could await that with raising photons number the Fock states will be included in the system dynamics. However, by assuming $\max(|\sigma|, |\mu|) << \chi_a, \chi_b$ permits us to use nonlinear quantum method of the scissors extensively discussed in Leoński and Tanaś (1994); Miranowicz

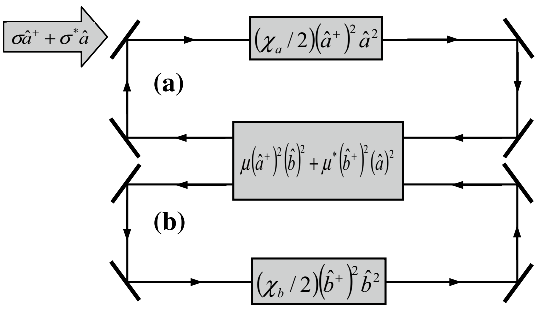

Fig. 1 The model of a nonlinear coupling coupler pumped in one mode

et al. 1996; Leoński 1997; Leoński et al. 1997; Leoński and Kowalewska-Kudłaszyk 2011; Leoński and Miranowicz 2001; Leoński and Miranowicz 2004). In consequence, we can truncate the wave function (2) to the wave function depicting only the progression of three resonant states $|2\rangle_a|0\rangle_b$, $|1\rangle_a|2\rangle_b$ and $|0\rangle_a|2\rangle_b$ with the form as (Le Duc and Cao Long 2016)

$$|\psi(t)\rangle_{cut} = c_{20}^{(jk)}(t)|2\rangle_a|0\rangle_b + c_{12}^{(jk)}(t)|1\rangle_a|2\rangle_b + c_{02}^{(jk)}(t)|0\rangle_a|2\rangle_b, \tag{3}$$

in which $j = 0, 1, 2$ and $k = 0, 2$ are the cavity modes indication, which are originally in states $|j\rangle_a|k\rangle_b$. By employing equation of the Schrödinger, the equations set for three amplitudes of the probability in the closed form can be written as

$$\begin{aligned} i\frac{d}{dt}c_{20}^{(jk)}(t) &= 2\mu c_{02}^{(jk)}(t), \\ i\frac{d}{dt}c_{12}^{(jk)}(t) &= \sigma c_{02}^{(jk)}(t), \\ i\frac{d}{dt}c_{02}^{(jk)}(t) &= 2\mu^* c_{20}^{(jk)}(t) + \sigma^* c_{12}^{(jk)}(t). \end{aligned} \tag{4}$$

In experimental, the linear coupling strength between the mode a and the external excitation field always contains some component of the fluctuation, so it is modelled by white noise. The parameter σ is now assumed consist of two elements: a deterministic component and white noise in the following form

$$\sigma = \sigma_0 + \sigma(t), \tag{5}$$

in which σ_0 is a deterministic coherent element and $\sigma(t)$ is characterized by a randomly fluctuating stochastic process (white noise) with the properties as

$$\left\langle\left\langle \sigma(t)\sigma^*\left(t'\right)\right\rangle\right\rangle = d_0\delta\left(t - t'\right), \tag{6}$$

where the double brackets represent an average over the realisations ensemble of the $\sigma(t)$, d_0 is parameter that relates to the noisy element. Then the set of motion Eqs. (4) is represented in terms of following equation of the stochastic differential

$$\frac{dC}{dt} = \left[M_1 + x(t)M_2 + x^*(t)M_3\right]C, \tag{7}$$

here C and M_1, M_2, M_3 are a time vector function and matrices of the constant, respectively. As known from the theory of the stochastic process of the multiplicative, the function $\langle\langle C\rangle\rangle$ suits the following equation:

$$\frac{d}{dt}\langle\langle C\rangle\rangle = \left[M_1 + d_0\left\{M_2, M_3\right\}/2\right]\langle\langle C\rangle\rangle, \tag{8}$$

where $\left\{M_2, M_3\right\}$ is the anticommutator of M_2 and M_3.

Then the set of motion equations for probability amplitudes become the stochastic differential equations. For simplicity, we assume that the parameters σ and μ are real and equal. From there, by applying formula (8) to the Eqs. (4), we achieve the stochastic averages equations with the form without double brackets as

$$\begin{aligned} i\frac{d}{dt}c_{20}^{(jk)}(t) =&2d_0c_{20}^{(jk)}(t) + d_0c_{12}^{(jk)}(t) + 2\sigma_0c_{02}^{(jk)}(t),\\ i\frac{d}{dt}c_{12}^{(jk)}(t) =&d_0c_{20}^{(jk)}(t) + \frac{d_0}{2}c_{12}^{(jk)}(t) + \sigma_0c_{02}^{(jk)}(t),\\ i\frac{d}{dt}c_{02}^{(jk)}(t) =&2\sigma_0c_{20}^{(jk)}(t) + \sigma_0c_{12}^{(jk)}(t) + \frac{5d_0}{2}c_{02}^{(jk)}(t). \end{aligned} \tag{9}$$

We can easily see that when $d_0 = 0$, Eqs. (9) become Eqs. (4). Then obtainable result is becoming exactly the one found by Kowalewska-Kudłaszyk et al. in Kowalewska-Kudłaszyk and Leoński (2006). By assuming that at the initial time, no photon is inside mode a and both photons are inside mode b ($|\psi(t=0)\rangle_{cut} = |0\rangle_a|2\rangle_b$), the solutions of Eqs. (9) were written as follows:

$$\begin{aligned} c_{20}^{(02)}(t) =& -\frac{2i\sqrt{5}}{5}\left\{\sin\left(\sqrt{5}\sigma_0t\right)\left[\cos\left(\frac{5d_0}{2}t\right) + i\sin\left(\frac{5d_0}{2}t\right)\right]\right\},\\ c_{12}^{(02)}(t) =& -\frac{i\sqrt{5}}{5}\left\{\sin\left(\sqrt{5}\sigma_0t\right)\left[\cos\left(\frac{5d_0}{2}t\right) + i\sin\left(\frac{5d_0}{2}t\right)\right]\right\},\\ c_{02}^{(02)}(t) =& \cos\left(\sqrt{5}\sigma_0t\right)\left[\cos\left(\frac{5d_0}{2}t\right) - i\sin\left(\frac{5d_0}{2}t\right)\right]. \end{aligned} \tag{10}$$

And when the time is zero, one photon is inside mode a and two photons are inside mode b, i.e., $c_{20}^{(12)}(0) = c_{02}^{(12)}(0) = 0$ and $c_{12}^{(12)}(0) = 1$ ($|\psi(t=0)\rangle_{cut} = |1\rangle_a|2\rangle_b$), we obtain the solutions of Eqs. (9) with the following form:

$$\begin{aligned} c_{20}^{(12)}(t) =& \frac{2}{5}\left\{\cos\left(\sqrt{5}\sigma_0t\right)\left[\cos\left(\frac{5d_0}{2}t\right) - i\sin\left(\frac{5d_0}{2}t\right)\right] - 2\right\},\\ c_{12}^{(12)}(t) =& \frac{1}{5}\left\{\cos\left(\sqrt{5}\sigma_0t\right)\left[\cos\left(\frac{5d_0}{2}t\right) - i\sin\left(\frac{5d_0}{2}t\right)\right] + 4\right\},\\ c_{02}^{(12)}(t) =& -\frac{i\sqrt{5}}{5}\left\{\sin\left(\sqrt{5}\sigma_0t\right)\left[\cos\left(\frac{5d_0}{2}t\right) + i\sin\left(\frac{5d_0}{2}t\right)\right]\right\}. \end{aligned} \tag{11}$$

On the other hand, by supposing that when the time is zero, both photons are inside mode a and no photon is inside mode b, namely $c_{20}^{(20)}(0) = 1$ and $c_{12}^{(20)}(0) = c_{02}^{(20)}(0) = 0$ ($|\psi(t=0)\rangle_{cut} = |2\rangle_a|0\rangle_b$). Then their solutions have the form as:

$$\begin{aligned} c_{20}^{(20)}(t) =& \frac{2}{5}\left\{2\cos\left(\sqrt{5}\sigma_0t\right)\left[\cos\left(\frac{5d_0}{2}t\right) - i\sin\left(\frac{5d_0}{2}t\right)\right] + 1\right\},\\ c_{12}^{(20)}(t) =& \frac{2}{5}\left\{\cos\left(\sqrt{5}\sigma_0t\right)\left[\cos\left(\frac{5d_0}{2}t\right) - i\sin\left(\frac{5d_0}{2}t\right)\right] - 1\right\},\\ c_{02}^{(20)}(t) =& -\frac{2i\sqrt{5}}{5}\left\{\sin\left(\sqrt{5}\sigma_0t\right)\left[\cos\left(\frac{5d_0}{2}t\right) + i\sin\left(\frac{5d_0}{2}t\right)\right]\right\}, \end{aligned} \tag{12}$$

Next, we going to examine the entropy of entanglement and the probabilities for existence of the system in the Bell-like states for the different values of parameter d_0 in the next section.

3 The creation of Bell-like states

The evolution of the system can be depicted in the form the qutrit-qubit system, one is able to wait that for the model considered here, the maximally entangled states can also be created. To examine this phenomenon, we graph the probabilities of the states in our system for $|\psi(t=0)\rangle_{cut} = |0\rangle_a|2\rangle_b$, $|\psi(t=0)\rangle_{cut} = |1\rangle_a|2\rangle_b$ and $|\psi(t=0)\rangle_{cut} = |2\rangle_a|0\rangle_b$ that are shown in Fig. 2. The probabilities $P_2^{(20)}$ become exactly the same as those in Duc and Cao Long (2016). We can see that the state pairs probabilities $|2\rangle_a|0\rangle_b$ and $|0\rangle_a|2\rangle_b$ ($P_2^{(20)}$), $|2\rangle_a|0\rangle_b$ and $|1\rangle_a|2\rangle_b$ ($P_2^{(12)}$) as well as $|2\rangle_a|0\rangle_b$ and $|0\rangle_a|2\rangle_b$ ($P_2^{(02)}$) intersect at approximate values equal to 0.5. Moreover, other combinations of the pure states in Fig. 2, which may play some role in the system progress such as the maximally entangled states including $|2\rangle_a|0\rangle_b$ and $|1\rangle_a|2\rangle_b$ could be checked. We can show that these states pair intersect with a probability value equal to 0.1. Thus, the system that is considered to be almost in the pure state $|0\rangle_a|2\rangle_b$ for these time moments. In addition, these probabilities turn into approximate to 0.5 for certain other time moments, even if they do not intersect. These mean that the maximally entangled states could be created in our system. Hence for the states of Bell's basis, the wave function can express in the form:

$$|\psi\rangle = \sum_{i=1}^{6} b_{i3N}^{(jk)} \left| B_{i3N}^{(jk)} \right\rangle, \tag{13}$$

in which $\left| B_{i3N}^{(jk)} \right\rangle$ are Bell-like states. These states may be displayed as functions of the Fock states:

$$\begin{aligned}
\left|B_{13N}^{(jk)}\right\rangle &= \frac{1}{\sqrt{2}}\left(|2\rangle_a|0\rangle_b + i|0\rangle_a|2\rangle_b\right)^{(jk)}, & \left|B_{23N}^{(jk)}\right\rangle &= \frac{1}{\sqrt{2}}\left(|2\rangle_a|0\rangle_b - i|0\rangle_a|2\rangle_b\right)^{(jk)},\\
\left|B_{33N}^{(jk)}\right\rangle &= \frac{1}{\sqrt{2}}\left(|2\rangle_a|0\rangle_b + i|1\rangle_a|2\rangle_b\right)^{(jk)}, & \left|B_{43N}^{(jk)}\right\rangle &= \frac{1}{\sqrt{2}}\left(|2\rangle_a|0\rangle_b - i|1\rangle_a|2\rangle_b\right)^{(jk)},\\
\left|B_{53N}^{(jk)}\right\rangle &= \frac{1}{\sqrt{2}}\left(|2\rangle_a|0\rangle_b + |1\rangle_a|2\rangle_b\right)^{(jk)}, & \left|B_{63N}^{(jk)}\right\rangle &= \frac{1}{\sqrt{2}}\left(|2\rangle_a|0\rangle_b - |1\rangle_a|2\rangle_b\right)^{(jk)}.
\end{aligned} \tag{14}$$

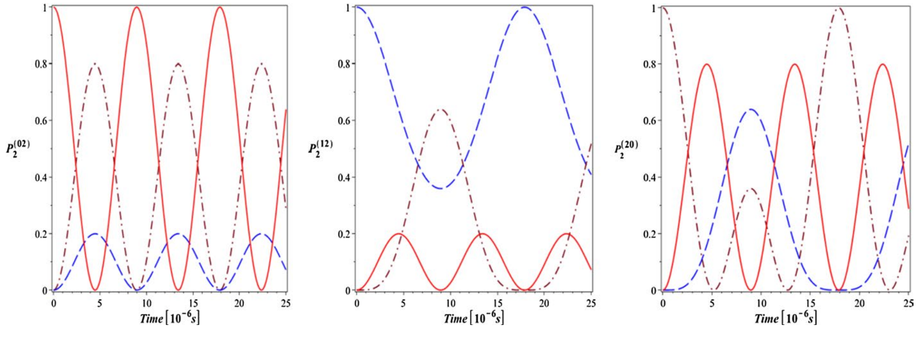

Fig. 2 The probabilities to the system exist in states $|0\rangle_a|2\rangle_b$ (solid line), $|1\rangle_a|2\rangle_b$ (dashed line), and $|2\rangle_a|0\rangle_b$ (dashed-dotted line) for $d_0 = 0$ and $\sigma_0 = 5\pi \times 10^4$ rad/s with $|\psi(t=0)\rangle_{cut} = |0\rangle_a|2\rangle_b$ ($P_2^{(02)}$), $|\psi(t=0)\rangle_{cut} = |1\rangle_a|2\rangle_b$ ($P_2^{(12)}$) and $|\psi(t=0)\rangle_{cut} = |2\rangle_a|0\rangle_b$ ($P_2^{(20)}$)

Thus, from Eqs. (3) and (14) we can find the coefficients $b_{i3N}^{(jk)}$ as follows

$$\begin{aligned} b_{13N}^{(jk)} &= \frac{1}{\sqrt{2}}\left(c_{20}^{(jk)}(t) - ic_{02}^{(jk)}(t)\right), \quad b_{23N}^{(jk)} = \frac{1}{\sqrt{2}}\left(c_{20}^{(jk)}(t) + ic_{02}^{(jk)}(t)\right), \\ b_{33N}^{(jk)} &= \frac{1}{\sqrt{2}}\left(c_{20}^{(jk)}(t) - ic_{12}^{(jk)}(t)\right), \quad b_{43N}^{(jk)} = \frac{1}{\sqrt{2}}\left(c_{20}^{(jk)}(t) + ic_{12}^{(jk)}(t)\right), \\ b_{53N}^{(jk)} &= \frac{1}{\sqrt{2}}\left(c_{20}^{(jk)}(t) + c_{12}^{(jk)}(t)\right), \quad b_{63N}^{(jk)} = \frac{1}{\sqrt{2}}\left(c_{20}^{(jk)}(t) - c_{12}^{(jk)}(t)\right). \end{aligned} \tag{15}$$

The capacity for creation of the maximally entangled states can be depicted by the entropy of Shannon S with respect to the coefficients of squared Schmidt $\lambda_{i3N}^{(jk)}$ (Vedral 2002) in the following form

$$E_{3N}^{(jk)}(t) = S\left(\lambda_{i3N}^{(jk)}\right) = -\sum_i \lambda_{i3N}^{(jk)} \log_2 \lambda_{i3N}^{(jk)} = -\lambda_{13N}^{(jk)} \log_2 \lambda_{13N}^{(jk)} - \lambda_{23N}^{(jk)} \log_2 \lambda_{23N}^{(jk)}, \tag{16}$$

in which $\lambda_{13N}^{(jk)} = \left|c_{20}^{(jk)}\right|^2$ and $\lambda_{23N}^{(jk)} = \left|c_{02}^{(jk)}\right|^2 + \left|c_{12}^{(jk)}\right|^2$. The value of entropy of entanglement modifies from zero to 1 ebit. It has as value as 1 ebit for the maximally states of the entangled and has as value as zero for the discrete states.

From the system of Eqs. (10), it is easy to see that the progression of entangled entropy and Bell-like states do not depend on the parameter related to the noisy element d_0 for the initial state case $|\psi(t=0)\rangle_{cut} = |0\rangle_a |2\rangle_b$. This shows that for the initial state where mode a contains no photons, and mode b contains both photons, the fluctuation of the external field pumped into mode a does not affect to the progression of the system under consideration. This means that in this case the coupling of the modes is very stable, not affected by the fluctuation of the coupling fields. Therefore, in this case we do not need to examine the dependence of entangled entropy and Bell-like states on parameter d_0.

The progress of entanglement entropies $E_{3N}^{(20)}$ and $E_{3N}^{(12)}$ are shown in Fig. 3. These results show that when $d_0 = 0$, $E_{3N}^{(20)}$ is exactly the same as those in Kowalewska-Kudłaszyk and Leoński (2006). $E_{3N}^{(20)}$ and $E_{3N}^{(12)}$ modify according to the time period and for the case when

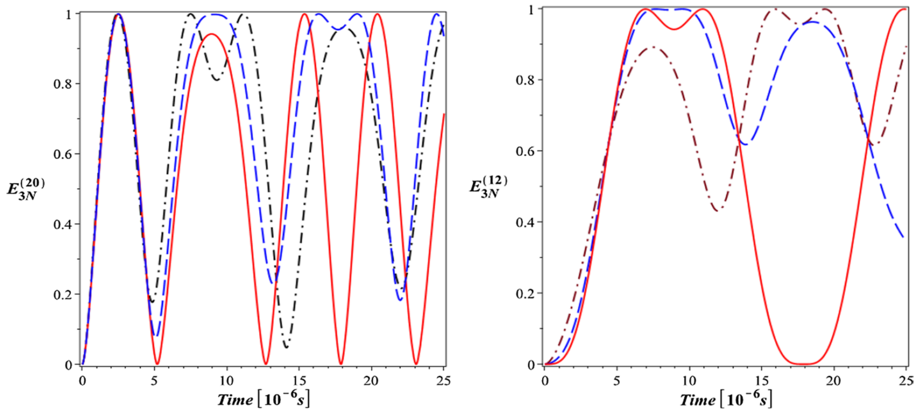

Fig. 3 The progress of entanglement entropies (ebits unit) $E_{3N}^{(20)}$ and $E_{3N}^{(12)}$ in the nonlinear coupling coupler that pumps by external field for single mode with $\sigma_0 = 5\pi \times 10^4$ rad/s. Solid line: $d_0 = 0$. Dashed line: $d_0 = 1,25\pi \times 10^4$ rad/s. Dashed-dotted line: $d_0 = 2,5\pi \times 10^4$ rad/s

Bell-like states are maximally entangled states, they have as values as 1 ebit whereas for separable states, they have as values as zero. Moreover, Fig. 3 shows that apart from second maximum of $E_{3N}^{(20)}$, other maxima are exactly equal to unity, which means this system may be studied like the source of maximally entangled states. Consequently, the entropies of entanglement and the maximally entangled states of our system change significantly for different initial conditions. When $d_0 \neq 0$, the entropies of entanglement $E_{3N}^{(20)}$ and $E_{3N}^{(12)}$ also change according to period of time. When the parameter d_0 increases, the second maximum of $E_{3N}^{(20)}$ also increases approximately equal to unity and then it splits into two lines while the next two maxima of $E_{3N}^{(20)}$ and the first two maxima of $E_{3N}^{(12)}$ decrease and is incorporated into a line. Thus, the entropy of entanglement progresses with time, when this maximum is reduced, the other maxima increased approximately equal to unity for different values of the parameter d_0. It means our system may also be created the maximally entangled states when d_0 is present. In addition, all minima of the $E_{3N}^{(20)}$ and $E_{3N}^{(12)}$ are non-zero as d_0 increases. It means that the states of our system in this case can always be in the entangled states. Furthermore, the intensity and maximum position of the entropies of entanglement also change with time as parameter d_0 increases.

Figures from 4 to 9 express the probabilities for existence of our system in the Bell-like states with the initial states $|\psi(t=0)\rangle_{cut} = |2\rangle_a|0\rangle_b$ and $|\psi(t=0)\rangle_{cut} = |1\rangle_a|2\rangle_b$ and the different values of parameter those relate to the noisy element d_0.

When d_0 is absent ($d_0 = 0$), the obtained results correspond to solid lines in Figs. 4, 5, 6, 7, 8, 9. The values of the highest maxima of the probabilities corresponding to the states $\left|B_{13N}^{(20)}\right\rangle$ (Fig. 4), $\left|B_{23N}^{(20)}\right\rangle$ (Fig. 5), $\left|B_{53N}^{(20)}\right\rangle$ (Fig. 8) and $\left|B_{63N}^{(12)}\right\rangle$ (Fig. 9) are approximately equal to unity. These mean that the maximally entangled states are created in our system while the maximally entangled states are not generated in the system for the states $\left|B_{13N}^{(12)}\right\rangle$ (Fig. 4), $\left|B_{23N}^{(12)}\right\rangle$ (Fig. 5), $\left|B_{33N}^{(20)}\right\rangle$ and $\left|B_{33N}^{(12)}\right\rangle$ (Fig. 6), $\left|B_{43N}^{(20)}\right\rangle$ and $\left|B_{43N}^{(12)}\right\rangle$ (Fig. 7), $\left|B_{53N}^{(12)}\right\rangle$ (Fig. 8) and $\left|B_{63N}^{(20)}\right\rangle$ (Fig. 9).

For the case parameter d_0 is present ($d_0 \neq 0$), when d_0 increases, the largest maximum of the probabilities for existence of the system in states $\left|B_{13N}^{(20)}\right\rangle$ (Fig. 4), $\left|B_{23N}^{(20)}\right\rangle$ (Fig. 5), $\left|B_{53N}^{(20)}\right\rangle$ (Fig. 8) and $\left|B_{63N}^{(12)}\right\rangle$ (Fig. 9) decrease. In addition, the maximum probabilities of our

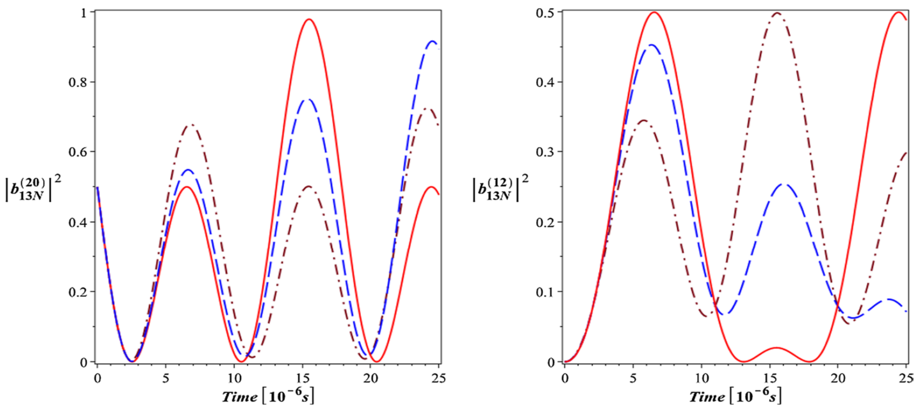

Fig. 4 The probabilities for existence of the system in Bell-like states $\left|B_{13N}^{(20)}\right\rangle$ and $\left|B_{13N}^{(12)}\right\rangle$ with $\sigma_0 = 5\pi \times 10^4$ rad/s. Solid line: $d_0 = 0$. Dashed line: $d_0 = 1,25\pi \times 10^4$ rad/s. Dashed-dotted line: $d_0 = 2,5\pi \times 10^4$ rad/s

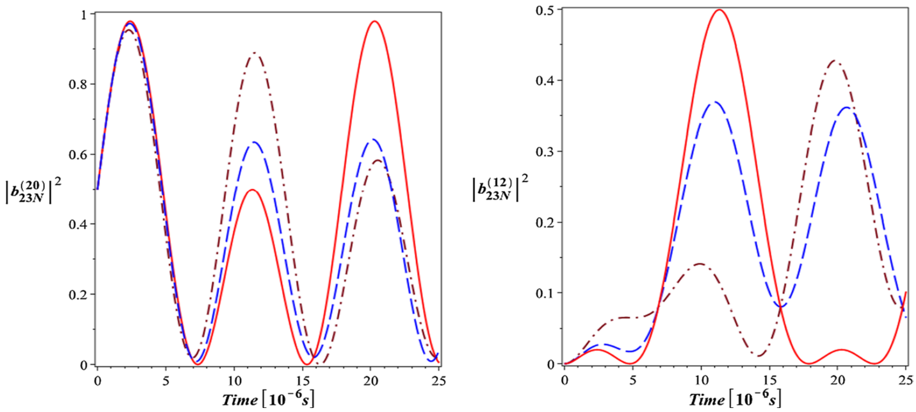

Fig. 5 The probabilities for existence of the system in Bell-like states $\left|B_{23N}^{(20)}\right\rangle$ and $\left|B_{23N}^{(12)}\right\rangle$ with $\sigma_0 = 5\pi \times 10^4$ rad/s. Solid line: $d_0 = 0$. Dashed line: $d_0 = 1,25\pi \times 10^4$ rad/s. Dashed-dotted line: $d_0 = 2,5\pi \times 10^4$ rad/s

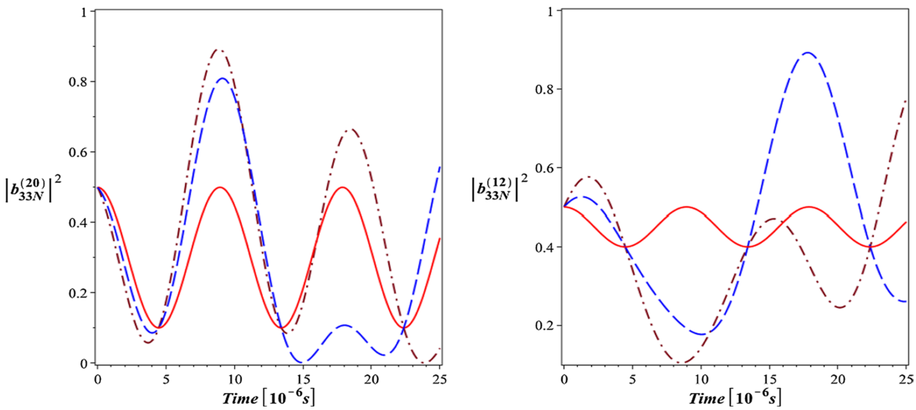

Fig. 6 The probabilities for existence of the system in Bell-like states $\left|B_{33N}^{(20)}\right\rangle$ and $\left|B_{33N}^{(12)}\right\rangle$ with $\sigma_0 = 5\pi \times 10^4$ rad/s. Solid line: $d_0 = 0$. Dashed line: $d_0 = 1,25\pi \times 10^4$ rad/s. Dashed-dotted line: $d_0 = 2,5\pi \times 10^4$ rad/s

system existing in the considered states with the values of 0.5 or less are enhanced in compared to the case when d_0 is absent. Consequently, our system may be generating maximally entangled states as the value of parameter d_0 is small, whereas for the d_0 value is large it cannot be created maximally entangled states. These results can be explained that the quantum interference in our system, which is considered to be weaker when the parameter d_0 increases. Additionally, when parameter d_0 increases, the minimum probabilities for existence of the system in states $\left|B_{33N}^{(12)}\right\rangle$ (Fig. 6), $\left|B_{43N}^{(12)}\right\rangle$ (Fig. 7) and $\left|B_{63N}^{(12)}\right\rangle$ (Fig. 8) are always greater than zero, which means these states are always entangled states.

Particularly, Figs. 6 and 7 show that when the parameter d_0 increases, some maximum of the probabilities for existence of the system in states $\left|B_{33N}^{(20)}\right\rangle$, $\left|B_{33N}^{(12)}\right\rangle$, $\left|B_{43N}^{(20)}\right\rangle$ and $\left|B_{43N}^{(12)}\right\rangle$ increase while the others decrease and the minima probabilities for existence of the system in these states decreases gradually to zero. It means that the probabilities for existence of the system in these entangled states are not stable.

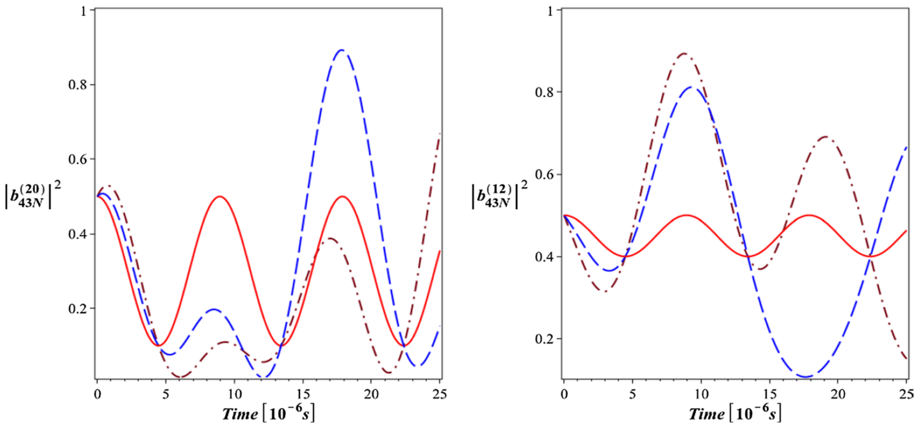

Fig. 7 The probabilities for existence of the system in Bell-like states $\left|B^{(20)}_{43N}\right\rangle$ and $\left|B^{(12)}_{43N}\right\rangle$ with $\sigma_0 = 5\pi \times 10^4$ rad/s. Solid line: $d_0 = 0$. Dashed line: $d_0 = 1,25\pi \times 10^4$ rad/s. Dashed-dotted line: $d_0 = 2,5\pi \times 10^4$ rad/s

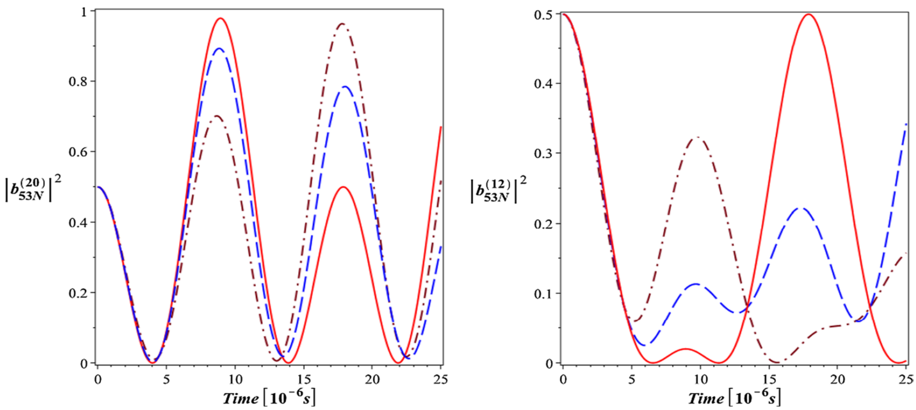

Fig. 8 The probabilities for existence of the system in Bell-like states $\left|B^{(20)}_{53N}\right\rangle$ and $\left|B^{(12)}_{53N}\right\rangle$ with $\sigma_0 = 5\pi \times 10^4$ rad/s. Solid line: $d_0 = 0$. Dashed line: $d_0 = 1,25\pi \times 10^4$ rad/s. Dashed-dotted line: $d_0 = 2,5\pi \times 10^4$ rad/s

4 Conclusions

In this paper, the nonlinear coupler with a nonlinear internal coupling is considered. This coupler interacts linearly with the external field that is modelled by while noise. By using the nonlinear quantum scissors method, the system progress grows closed in a set with three states $|2\rangle_a|0\rangle_b$, $|1\rangle_a|2\rangle_b$ and $|0\rangle_a|2\rangle_b$. We have shown that when the parameter d_0 is small, our system may also generate the maximally entangled states whereas the maximally entangled states cannot be created for large value of the parameter d_0. Moreover, the probabilities maximum positions for existence of the system in states of the maximal entanglement are changed with time and some states of the system are always entanglement states compared with the case d_0 is not present. Thus, the parameter that relates to the noisy element d_0 is a key parameter to controls the change of the intensity and maximum positions of Bell-like states.

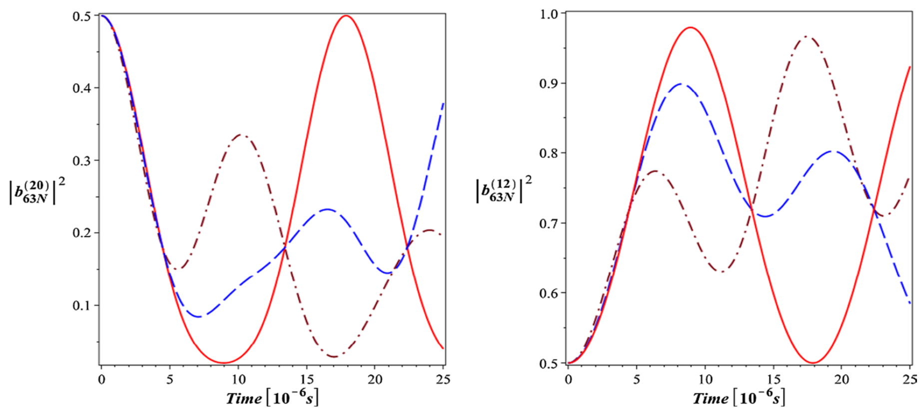

Fig. 9 The probabilities for existence of the system in Bell-like states $\left|B^{(20)}_{63N}\right\rangle$ and $\left|B^{(12)}_{63N}\right\rangle$ with $\sigma_0 = 5\pi \times 10^4$ rad/s. Solid line: $d_0 = 0$. Dashed line: $d_0 = 1,25\pi \times 10^4$ rad/s. Dashed-dotted line: $d_0 = 2,5\pi \times 10^4$ rad/s

Supplementary Information The online version contains supplementary material available at https://doi.org/10.1007/s11082-021-03416-7.

Acknowledgements The study was supported by The Youth Incubator for Science and Technology Programe, managed by Youth Development Science and Technology Center—Ho Chi Minh Communist Youth Union and Department of Science and Technology of Ho Chi Minh City, the contract number is "01/2020/HĐ-KHCNT-VƯ".

References

Barnett, S.M., Pegg, D.T.: Optical state truncation. Phys. Rev. A **60**, 4965–4973 (1999)

Cao Long, V., Trippenbach, M.: Photoelectron spectra induced by broad-band chaotic light. Zeitschrift Für Physik B **63**, 267–272 (1986)

Doan Quoc, K., Cao Long, V., Leoński, W.: A broad-band laser-driven double Fano system—photoelectron spectra. Phys. Scr. **86**, 045301 (2012a)

Doan Quoc, K., Cao Long, V., Leoński, W.: Electromagnetically induced transparency for Λ-like systems with a structured continuum and broad-band coupling laser. Phys. Scr. T **147**, 014008 (2012b)

Doan Quoc, K., Cao Long, V., Van Chu, L., Nguyen Thanh, V., Tran Thi, H., Ha Kim, Q., Nguyen Thi Hong, S.: Kerr nonlinear coupler and entanglement induced by broadband laser light. Photon. Lett. Pol. **8**, 64–66 (2016)

Doan Quoc, K., Cao Long, V., Van Chu, L., Huynh Vinh, P.: Electromagnetically induced transparency for Λ-like systems with degenerate autoionizing levels and a broadband coupling laser. Opt. Appl. **46**, 93–102 (2016)

Doan Quoc, K., Nguyen Ba, D., Thai Doan, T., Ho Quang, Q., Cao Long, V., Leoński, W.: Broadband laser-driven electromagnetically induced transparency in three-level systems with a double Fano continuum. J. Opt. Soc. Am. B **35**, 1536–1544 (2018)

Doan Quoc, K., Luong Thi, Tu.O., Van Chu, L., Nguyen Thi, D., Van Nguyen, H.: Generation of entangled states by a nonlinear coupler pumped in one mode induced by broadband laser. Opt. Quant. Electron. **52**, 1–12 (2020)

Einstein, A., Podolsky, B., Rosen, N.: Can quantum-mechanical description of physical reality be considered complete? Phys. Rev. **47**, 777–780 (1935)

Kalaga, J.K., Leoński, W.: Quantum steering borders in three-qubit systems. Quantum Inf. Process. **16**, 175 (2017)

Kalaga, J.K., Kowalewska-Kudłaszyk, A., Leoński, W., Barasiński, A.: Quantum correlations and entanglement in a model comprised of a short chain of nonlinear oscillators. Phys. Rev. A **94**, 032304 (2016)

Kalaga, J.K., Leoński, W., Peřina, J., Jr.: Einstein-Podolsky-Rosen steering and coherence in the family of entangled three-qubit states. Phys. Rev. A **97**, 042110 (2018)

Kowalewska-Kudłaszyk, A., Leoński, W.: Finite-dimensional states and entanglement generation for a nonlinear coupler. Phys. Rev. A **73**, 042318 (2006)

Kowalewska-Kudłaszyk, A., Leoński, W.: Nonlinear coupler operating on Werner-like states-entanglement creation, its enhancement, and preservation. J. Opt. Soc. Am. B **31**, 1290–1297 (2014)

Kowalewska-Kudłaszyk, A., Leoński, W., Perina, J., Jr.: Generalized Bell states generation in a parametrically excited nonlinear coupler. Phys. Scr. T **147**, 014016 (2012)

Kowalewska-Kudłaszyk, A., Leoński, W., Nguyen, T.D., Cao Long, V.: Kicked nonlinear quantum scissors and entanglement generation. Phys. Scr. T **160**, 014023 (2014)

Le Duc, V., Cao Long, V.: Entangled State creation by a nonlinear coupler pumped in two modes. Comp. Meth. Sci. Technol. **22**, 245–252 (2016)

Leoński, W.: Finite-dimensional coherent-state generation and quantum-optical nonlinear oscillator models. Phys. Rev. A **55**, 3874–3878 (1997)

Leoński, W., Kowalewska-Kudłaszyk, A.: Quantum scissors-finite-dimensional states engineering. Prog. Opt. **56**, 131–185 (2011)

Leoński, W., Miranowicz, A.: Quantum-optical states in finite-dimensional hilbert space. II state generation, pp. 195–213. Wiley, New York (2001)

Leoński, W., Miranowicz, A.: Kerr nonlinear coupler and entanglement. J. Opt. B Quantum Semiclass Opt. **6**, S37–S42 (2004)

Leoński, W., Tanaś, R.: Possibility of producing the one-photon state in a kicked cavity with a nonlinear Kerr medium. Phys. Rev. A **49**, R20–R23 (1994)

Leoński, W., Dyrting, S., Tanaś, R.: Fock states generation in a kicked cavity with a nonlinear medium. J. Mod. Opt. **44**, 2105–2123 (1997)

Luong Thi Tu, O., Doan Quoc, K., Van Chu, L., Hoang Minh, D., Nguyen Thi, D., Nguyen Thi, T., Ngo Xuan, C.: Generation of entangled states by a linear coupling coupler pumped in two modes induced by broadband laser. Optik **208**, 164028 (2020)

Miranowicz, A., Leoński, W.: Two-mode optical state truncation and generation of maximally entangled states in pumped nonlinear couplers. J. Phys. B at. Mol. Opt. Phys. **39**, 1683–1700 (2006)

Miranowicz, A., Leoński, W., Dyrting, S., Tanaś, R.: Quantum state engineering in finite-dimensional Hilbert space. Acta Phys. Slovaca **46**, 451–456 (1996)

Ozdemir, S.K., Miranowicz, A., Koashi, M., Imoto, N.: Quantum-scissors device for optical state truncation: a proposal for practical realization. Phys. Rev. A **64**, 063818 (2001)

Pegg, D.T., Phillips, L.S., Barnett, S.M.: Optical state truncation by projection synthesis. Phys. Rev. Lett. **81**, 1604–1606 (1998)

Peřinová, V., Lukš, A., Křepelka, J., Leoński, W., Peřina, J., Jr.: Kerr-like behaviour of second harmonic generation in the far-off resonant regime. Opt. Commun. **414**, 146–153 (2018)

Vedral, V.: The role of relative entropy in quantum information theory. Rev. Mod. Phys. **74**, 197–234 (2002)