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Original Article

Determined Dispersion Coefficient of ^{85}Rb Atom in the Y–Configuration

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Abstract: In this work, we derive analytical expression for the dispersion coefficient of ^{85}Rb atom for a weak probe laser beam induced by a strong coupling laser beam. Our results show possible ways to control dispersion coefficient by frequency detuning and of the coupling lasers. The results show that a Y-configuration appears two transparent window of the dispersion coefficient for the probe laser beam. The depth and width or position of these windows can be altered by changing the intensity or frequency detuning of the coupling laser fields.

Keywords: Electromagnetically induced transparency, dispersion coefficient.

1. Introduction

The manipulation of subluminal and superluminal light propagation in optical medium has attracted many attentions due to its potential applications during the last decades, such as controllable optical delay lines, optical switching [1], telecommunication [2], interferometry, optical data storage and optical memories quantum information processing, and so on [3]. The most important key to manipulate subluminal and superluminal light propagations lies in its ability to control the absorption and dispersion properties of a medium by a laser field [4, 5].

As we know that coherent interaction between atom and light field can lead to interesting quantum interference effects such as electromagnetically induced transparency (EIT) [6]. The EIT is a quantum interference effect between the probability amplitudes that leads to a reduction of resonant absorption for a weak probe light field propagating through a medium induced by a strong coupling light field. Basic configurations of the EIT effect are three-level atomic systems including the Λ-Ladder and V-

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type configurations. In each configuration, the EIT efficiency is different, in which the Λ-type configuration is the best, whereas the V-type configuration is the worst [7], therefore, the manipulation of light in each configuration are also different. To increase the applicability of this effect, scientists have paid attention to creating many transparent windows. One proposed option is to add coupling laser fields to further stimulate the states involved in the interference process. This suggests that we choose to use the analytical model to determine the dispersion coefficient for the *Y* configuration of the ^{85}Rb atomic system [8].

2. The density matrix equation

We consider a *Y*-configuration of ^{85}Rb atom as shown in Fig. 1. State $|1\rangle$ is the ground states of the level $5S_{1/2}$ (F=3). The $|2\rangle$, $|3\rangle$ and $|4\rangle$ states are excited states of the levels $5P_{3/2}$ (F'=3), $5D_{5/2}$ (F"=4) and $5D_{5/2}$ (F"=3) [8].

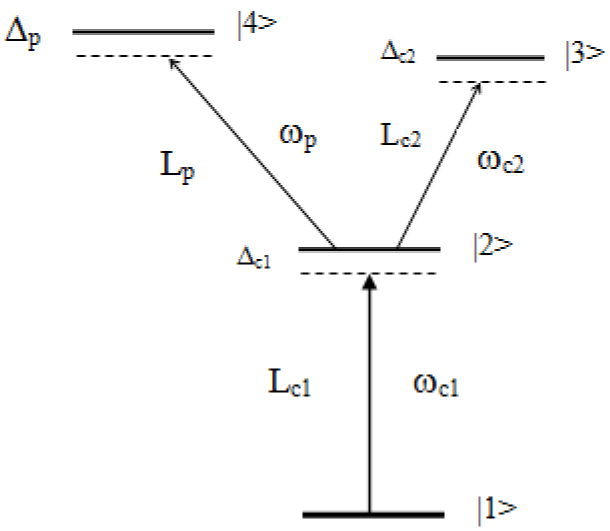

Fig 1. Four-level excitation of the Y- configuration.

Put this Y-configuration into three laser beams atomic frequency and intensity appropriate: a week probe laser L_p has intensity E_p with frequency ω_p applies the transition $|2\rangle \rightarrow |4\rangle$ and the Rabi frequencies of the probe $\Omega_p = \frac{\mu_{42} E_p}{\hbar}$; two strong coupling laser L_{c1} and L_{c2} couple the transition $|1\rangle \rightarrow |2\rangle$ and $|2\rangle \rightarrow |3\rangle$ the Rabi frequencies of the two coupling fields $\Omega_{c1} = \frac{\mu_{21} E_{c1}}{\hbar}$ and $\Omega_{c2} = \frac{\mu_{32} E_{c2}}{\hbar}$, where μ_{ij} is the electric dipole matrix element $|i\rangle \leftrightarrow |j\rangle$.

The evolution of the system, which is represented the density operator ρ is determined by the following Liouville equation [2]:

$$\frac{\partial\rho}{\partial t} = -\frac{i}{\hbar}[H,\rho] + \Lambda\rho \tag{1}$$

where, H represents the total Hamiltonian and $\Lambda\rho$ represents the decay part. Hamilton of the systerm can be written by matrix form:

$$H_0 = \hbar\omega_1|1\rangle\langle 1| + \hbar\omega_2|2\rangle\langle 2| + \hbar\omega_3|3\rangle\langle 3| + \hbar\omega_4|4\rangle\langle 4| \tag{2.2}$$

$$H_I = \frac{\hbar\Omega_p}{2}\left(|4\rangle\langle 2|e^{i\omega_p t} + |2\rangle\langle 4|e^{-i\omega_p t}\right) + \frac{\hbar\Omega_{c1}}{2}\left(|2\rangle\langle 1|e^{i\omega_{c1}t} + |1\rangle\langle 2|e^{-i\omega_{c1}t}\right) + \frac{\hbar\Omega_{c2}}{2}\left(|3\rangle\langle 2|e^{i\omega_{c2}t} + |2\rangle\langle 3|e^{-i\omega_{c2}t}\right) \tag{2.3}$$

In the framework of the semiclassical theory, the density matrix equations can be written as:

$$[\rho,H]_{44} = \frac{\hbar\Omega_p}{2}\left(e^{i\omega_p t}\rho_{42} - e^{-i\omega_p t}\rho_{24}\right) - \Gamma_{43}\rho_{44} \tag{3.1}$$

$$[\rho,H]_{41} = \frac{\hbar\Omega_{c1}}{2}e^{-i\omega_{c1}t}\rho_{42} - \frac{\hbar\Omega_p}{2}e^{-i\omega_p t}\rho_{21} + \hbar(\omega_1-\omega_4)\rho_{41} - \gamma_{41}\rho_{41} \tag{3.2}$$

$$[\rho,H]_{42} = \frac{\hbar\Omega_{c1}}{2}e^{i\omega_{c1}t}\rho_{41} + \frac{\hbar\Omega_{c2}}{2}e^{-i\omega_{c2}t}\rho_{43} + \frac{\hbar\Omega_p}{2}e^{-i\omega_p t}(\rho_{44}-\rho_{22}) + \hbar(\omega_2-\omega_4)\rho_{42} - \gamma_{42}\rho_{42} \tag{3.3}$$

$$[\rho,H]_{43} = \frac{\hbar\Omega_{c2}}{2}e^{i\omega_{c2}t}\rho_{42} - \frac{\hbar\Omega_p}{2}e^{-i\omega_p t}\rho_{23} + \hbar(\omega_3-\omega_4)\rho_{43} - \gamma_{43}\rho_{43} \tag{3.4}$$

$$[\rho,H]_{33} = \frac{\hbar\Omega_{c2}}{2}\left(e^{i\omega_{c2}t}\rho_{32} - e^{-i\omega_{c2}t}\rho_{23}\right) + \Gamma_{43}\rho_{44} - \Gamma_{32}\rho_{33} \tag{3.5}$$

$$[\rho,H]_{31} = \frac{\hbar\Omega_{c1}}{2}e^{-i\omega_{c1}t}\rho_{32} - \frac{\hbar\Omega_{c2}}{2}e^{-i\omega_{c2}t}\rho_{21} + \hbar(\omega_1-\omega_3)\rho_{31} - \gamma_{31}\rho_{31} \tag{3.6}$$

$$[\rho,H]_{32} = \frac{\hbar\Omega_{c1}}{2}e^{i\omega_{c1}t}\rho_{31} + \frac{\hbar\Omega_{c2}}{2}e^{-i\omega_{c2}t}(\rho_{33}-\rho_{22}) + \frac{\hbar\Omega_p}{2}e^{-i\omega_p t}\rho_{34} + \hbar(\omega_2-\omega_3)\rho_{32} - \gamma_{32}\rho_{32} \tag{3.7}$$

$$[\rho,H]_{34} = \frac{\hbar\Omega_p}{2}e^{i\omega_p t}\rho_{32} - \frac{\hbar\Omega_{c2}}{2}e^{-i\omega_{c2}t}\rho_{24} + \hbar(\omega_4-\omega_3)\rho_{34} - \gamma_{43}\rho_{34} \tag{3.8}$$

$$[\rho,H]_{22} = \frac{\hbar\Omega_{c1}}{2}\left(e^{i\omega_{c1}t}\rho_{21} - e^{-i\omega_{c1}t}\rho_{12}\right) + \frac{\hbar\Omega_{c2}}{2}\left(e^{-i\omega_{c2}t}\rho_{23} - e^{i\omega_{c2}t}\rho_{32}\right) + \frac{\hbar\Omega_p}{2}\left(e^{-i\omega_p t}\rho_{24} - e^{i\omega_p t}\rho_{42}\right) + \Gamma_{32}\rho_{33} - \Gamma_{21}\rho_{22} \tag{3.9}$$

$$[\rho,H]_{21} = \frac{\hbar\Omega_{c1}}{2}e^{-i\omega_{c1}t}(\rho_{22}-\rho_{11}) - \frac{\hbar\Omega_{c2}}{2}e^{i\omega_{c2}t}\rho_{31} - \frac{\hbar\Omega_p}{2}e^{i\omega_p t}\rho_{41} + \hbar(\omega_1-\omega_2)\rho_{21} - \gamma_{21}\rho_{21} \tag{3.10}$$

$$[\rho,H]_{23}=\frac{\hbar\Omega_{c2}}{2}e^{i\omega_{c2}t}\left(\rho_{22}-\rho_{33}\right)-\frac{\hbar\Omega_{c1}}{2}e^{-i\omega_{c1}t}\rho_{13}-\frac{\hbar\Omega_{p}}{2}e^{i\omega_{p}t}\rho_{43}$$
$$+\hbar\left(\omega_3-\omega_2\right)\rho_{23}-\gamma_{32}\rho_{23} \quad (3.11)$$

$$[\rho,H]_{24}=\frac{\hbar\Omega_{p}}{2}e^{i\omega_{p}t}(\rho_{22}-\rho_{44})-\frac{\hbar\Omega_{c1}}{2}e^{-i\omega_{c1}t}\rho_{14}-\frac{\hbar\Omega_{c2}}{2}e^{i\omega_{c2}t}\rho_{34}$$
$$+\hbar\left(\omega_4-\omega_2\right)\rho_{24}-\gamma_{42}\rho_{24} \quad (3.12)$$

$$[\rho,H]_{11}=\frac{\hbar\Omega_{c1}}{2}e^{i\omega_{c1}t}\left(\rho_{12}-\rho_{21}\right)+\Gamma_{21}\rho_{22} \quad (3.13)$$

$$[\rho,H]_{12}=\frac{\hbar\Omega_{c1}}{2}e^{i\omega_{c1}t}\left(\rho_{11}-\rho_{22}\right)+\frac{\hbar\Omega_{c2}}{2}e^{-i\omega_{c2}t}\rho_{13}+\frac{\hbar\Omega_{p}}{2}e^{-i\omega_{p}t}\rho_{14}$$
$$+\hbar\left(\omega_2-\omega_1\right)\rho_{12}-\gamma_{21}\rho_{12} \quad (3.14)$$

$$[\rho,H]_{13}=\frac{\hbar\Omega_{c2}}{2}e^{i\omega_{c2}t}\rho_{12}-\frac{\hbar\Omega_{c1}}{2}e^{i\omega_{c1}t}\rho_{23}+\hbar\left(\omega_3-\omega_1\right)\rho_{13}-\gamma_{31}\rho_{13} \quad (3.15)$$

$$[\rho,H]_{14}=\frac{\hbar\Omega_{p}}{2}e^{i\omega_{p}t}\rho_{12}-\frac{\hbar\Omega_{c1}}{2}e^{i\omega_{c1}t}\rho_{24}+\hbar\left(\omega_4-\omega_1\right)\rho_{14}-\gamma_{41}\rho_{14} \quad (3.16)$$

In addition, we suppose the initial atomic system is at a level $|2\rangle$ therefore, $\rho_{11}\approx\rho_{33}\approx\rho_{44}\approx 0, \rho_{22}=1$ and solve the density matrix equations under the steady-state condition by setting the time derivatives to zero:

$$\frac{d\rho}{dt}=0 \quad (4)$$

We consider the slow variation and put: $\rho_{43}=\tilde{\rho}_{43}e^{-i(\omega_p-\omega_{c2})t}$, $\rho_{42}=\tilde{\rho}_{42}e^{-i\omega_p t}$, $\rho_{41}=\tilde{\rho}_{41}e^{-i\left(\omega_p+\omega_{c1}\right)t}$, $\rho_{32}=\tilde{\rho}_{32}e^{-i\omega_{c2}t}$, $\rho_{31}=\tilde{\rho}_{31}e^{-i(\omega_{c1}+\omega_{c2})t}$, $\rho_{21}=\tilde{\rho}_{21}e^{-i\omega_{c1}t}$. Therefore, the equations (3.2), (3.3) and (3.4) are rewriten:

$$0=\frac{i\Omega_{c1}}{2}\tilde{\rho}_{42}-\frac{i\Omega_{p}}{2}\rho_{21}+[i(\Delta_{c1}+\Delta_p)-\gamma_{41}]\tilde{\rho}_{41} \quad (5.1)$$

$$0=\frac{i\Omega_{c1}}{2}\tilde{\rho}_{41}+\frac{i\Omega_{c2}}{2}\tilde{\rho}_{43}+\frac{i\Omega_{p}}{2}(\rho_{44}-\rho_{22})+(i\Delta_p-\gamma_{42})\tilde{\rho}_{42} \quad (5.2)$$

$$0=\frac{i\Omega_{c2}}{2}\tilde{\rho}_{42}-\frac{i\Omega_{p}}{2}\tilde{\rho}_{23}+[i(\Delta_p-\Delta_{c2})-\gamma_{43}]\tilde{\rho}_{43} \quad (5.3)$$

where, the frequency detuning of the probe and L_{c1}, L_{c2} coupling lasers from the relevant atomic transitions are respectively determined by $\Delta_p=\omega_p-\omega_{42}$, $\Delta_{c1}=\omega_{c1}-\omega_{21}$.

Because of $\Omega_p << \Omega_{c1}$ and Ω_{c2} so that we ignore the term $\frac{i\Omega_p}{2}\tilde{\rho}_{21}$ and $\frac{i\Omega_p}{2}\tilde{\rho}_{23}$ in the equations (4) and (5). We slove the equations (4) – (5):

$$\tilde{\rho}_{42} = \frac{-i\Omega_p / 2}{\gamma_{42} - i\Delta_p + \frac{\Omega_{c1}^2 / 4}{\gamma_{41} - i(\Delta_p + \Delta_{c1})} + \frac{\Omega_{c2}^2 / 4}{\gamma_{43} - i(\Delta_p - \Delta_{c2})}} \quad (6)$$

3. Dispersion coefficient

We start from the susceptibility of atomic medium for the probe light that is determined by the following relation:

$$\chi = -2\frac{Nd_{21}}{\varepsilon_0 E_p}\rho_{21} = \chi' + i\chi'' \quad (7)$$

The dispersion coefficient n of the atomic medium for the probe beam is determined through the imaginary part of the linear susceptibility (7):

$$n = 1 + \frac{1}{2}\chi' = 1 - \frac{N\mu_{42}^2}{\Omega_p \hbar \varepsilon_0}\mathrm{Re}(\tilde{\rho}_{42}) \; 8)$$

We considere the case of ^{85}Rb atom: γ_{42} = 3MHz, γ_{41} = 0.3MHz and γ_{43} = 0.03MHz, the atomic density $N = 10^{17}/\mathrm{m}^3$. The electric dipole matrix element is $d_{42} = 2.54.10^{-29}$ Cm, dielectric coefficient $\varepsilon_0 = 8.85.10^{-12}$ F/m, $\hbar = 1.05.10^{-34}$ J.s, and frequency of probe beam $\omega_p = 3.84.10^8$ MHz.

Fixed frequency Rabi of coupling laser beam L_{c1} in value Ω_{c1} = 16MHz (correspond to the value that when there is no laser L_{c2} then the transparency of the probe beam near 100%) and the frequency coincides with the frequency of the transition $|1\rangle \leftrightarrow |2\rangle$, it means $\Delta_{c1} = 0$. We consider the case of the frequency detuning of the coupling laser beam L_{c2} is Δ_{c2} = 10MHz and plot a three-dimensional graph of the dispersion coefficient n at the intensity of the coupling laser beam L_{c2} (Rabi frequency Ω_{c2}) and the frequency detuning of the probe laser beam L_p, the result is shown in Fig 2.

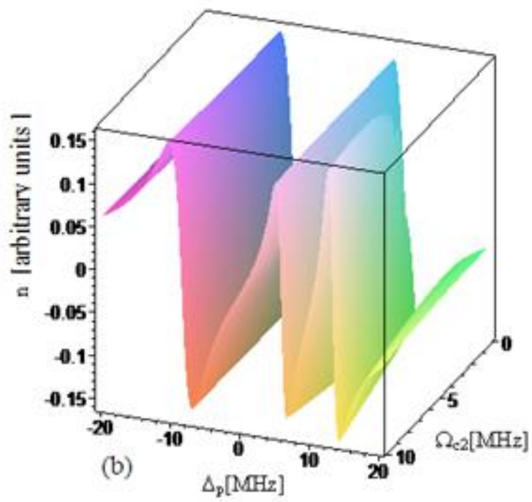

Fig 2**.** Three-dimensional graph of the dispersion coefficient n according **to** Δ_p and Ω_{c2} with Δ_{c1} = 0 MHz

As shown in Fig 2, we see that when there is no coupling laser beam, it makes L_{c2} (Ω_{c2} = 0), only a normal dispersion domain (in anomalous dispersion domain for a two-level system) corresponds to the transparent window above absorbing curve. In the presence of the coupling laser beam L_{c2} (with the selected frequency detuning Δ_{c2} = 10MHz) and gradually increasing the Rabi frequency Ω_{c2}, we see the second normal dispersion domain corresponding to the transparent window of the second on the absorption current, the spectral width of this region also increases with the increase of Ω_{c2} but the slope of this curve is reduced. To be more specific, we plot a two-dimensional graph of Fig 3 with some specific values of Rabi frequency Ω_{c2}.

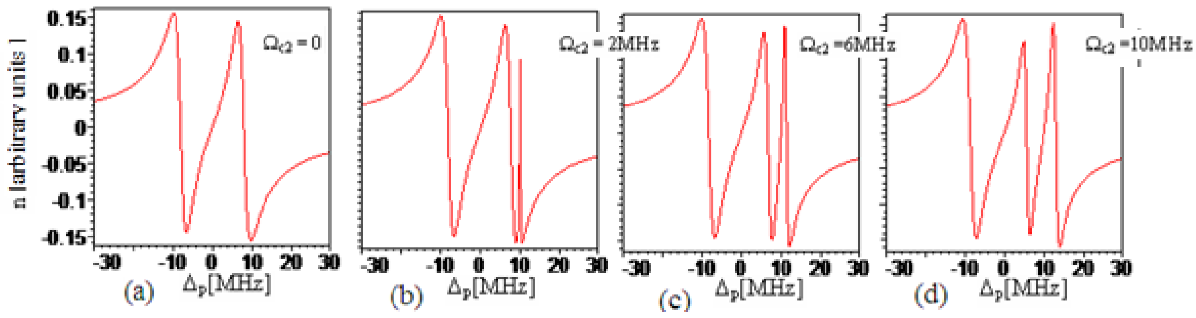

Fig 3. Two-dimensional graph of the dispersion coefficient *n* according to Δ_p with Ω_{c1} = 16MHz, Δ_{c1} = 0 MHz and Δ_{c2} = 10MHz.

Fig 3 is a graph of dispersion coefficient when there are no coupling laser fields, ie a two-level system. We found that the maximum absorbance at the resonance frequency and the anomaly dispersion region has not yet appeared the normal dispersion domain, the dispersion coefficient has a very small value at the adjacent resonant frequency of the probe beam.

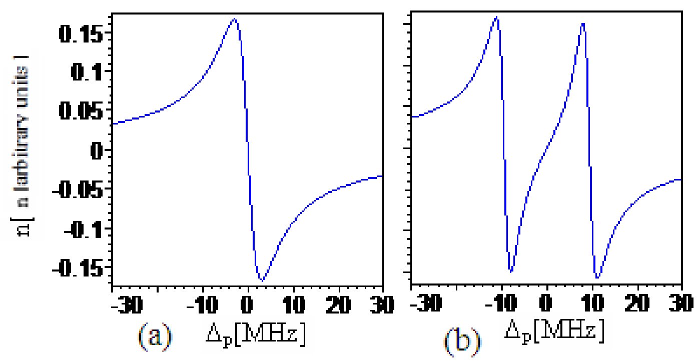

Fig 4. Two-dimensional graph of the dispersion coefficient *n* according to Δ_p with Ω_{c1} = 16 MHz (a), Ω_{c2} =10MHz (b) when frequency detuning Δ_{c1} = Δ_{c2} = 0.

Fig 4 is a graph of the dispersion coefficients when there are simultaneous presence of three laser beams (a probe laser and two coupling lasers), in which the control laser beams are tuned to resonate with the corresponding shift, ie $\Delta_{c1} = \Delta_{c2} = 0$. We see two transparent windows overlap each other (ie only one transparent window on the absorption curve) and therefore only one normal dispersion often corresponds.

4. Conclusions

In the framework of the semi-classical theory, we have cited the density matrix equation for the ^{85}Rb atomic system in the *Y*-configuration under the simultaneous effects of two laser probe and coupling beams. Using approximate rotational waves and approximate electric dipoles, we have found solutions in the form of analytic for the dispersion coefficient of atoms when the probe beam has a small intensity compared to the coupling beams. Drawing the dispersion coefficient expression will facilitate future research applications. Consequently, we investigated the absorption of the detector beam according to the intensity of the coupling beam Ω_{c1}, Ω_{c2} and the detuning of the probe beam Δ_p. The results show that a Y-configuration appears two transparent window for the probe laser beam. The depth and width or position of these windows can be altered by changing the intensity or frequency detuning of the coupling laser fields.

References

[1] R.W. Boyd, Slow and fast light: fundamentals and applications, J. Mod. Opt. 56 (2009) 1908–1915.

[2] J. Javanainen, Effect of State Superpositions Created by Spontaneous Emission on Laser-Driven Transitions, Europhys. Lett. 17 (1992) 407.

[3] M. Fleischhauer, I. Mamoglu, and J. P. Marangos, Electromagnetically induced transparency: optics in coherent media, Rev. Mod. Phys. 77 (2005) 633-673.

[4] L.V. Doai, D.X. Khoa, N.H. Bang, EIT enhanced self-Kerr nonlinearity in the three-level lambda system under Doppler broadening, Phys. Scr. 90 (2015) 045502.

[5] D.X. Khoa, P.V. Trong, L.V. Doai, N.H. Bang, Electromagnetically induced transparency in a five-level cascade system under Doppler broadening: an analytical approach, Phys, Scr. 91 (2016) 035401.

[6] K.J. Boller, A. Imamoglu, S.E. Harris, Observation of electromagnetically induced transparency, Phys. Rev. Lett. 66 (1991) 2593.

[7] S. Sena, T.K. Dey, M.R. Nath, G. Gangopadhyay, Comparison of Electromagnetically Induced Transparency in lambda, vee and cascade three-level systems, J. Mod. Opt. 62 (2014) 166-174.

[8] Daniel Adam Steck, ^{85}Rb D Line Data: http://steck.us/alkalidata (2013) (accessed 20 September 2013)