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Reliability assessment of Buckling Strength for Battened Built-up Columns steel considering shear deformations

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Abstract. Battened built-up columns are widely used in steel construction especially when the effective lengths are great and the compression forces light. The buckling strength of this structure depends hugely on the material properties, geometry dimensions and also the flexural deformations both of the chords so the battens must be considered in order to derive the shear stiffness. The input parameters are potentially random sources. This research presents an assessment of the safety for Buckling Strength for Battened Built-up Columns steel considering shear deformations. Reliability of the structure is evaluated using Monte Carlo simulation method. Effects of input parameters are also investigated.

1. Introduction

Nowadays, Battened built-up columns are more and more used in the civil and industrial domain. The buckling design of this type of steel column depends on many parameters and possesses latent errors. Thus, it is an interesting topic to the researchers all over the world [1, 2]. However, in the lifetime with latent randomness or the variation of external loads, in some cases, push the structure to the dangerous state. Thus, the reliability assessment of the structure is necessary.

Concerning the reliability assessment is also topic interesting to the researchers all over the world. In 1990, Kiureghian et al. assessed the reliability of the steel frame under the dynamic loads generated from the earthquake El Centro in 1940 using β probability index method [3]. The same method was used in [4] by Hong et al. to study the steel frame according to the plastic limit state. Reliability of the steel structure under the corrosion was considered by Robert and Melchers in [5]. Hadianfard and Razani used the Monte Carlo simulation method to investigate the steel frame with flexible beam-column joint [6]. The rotational stiffness and the moment resisting of the connection were referred in [7] of Chen et al. This model of flexible joint, according to recommendation of Aleksander in [8], is not correct. This author proposed then a novel model based on the empirical tests and the Eurocode 3.

Concerning the buckling of the steel column, some studies were carried out. In 1961, Timoshenko and Gere proposed a numerical solution in the similar form of the Euler's solution with an adjustment factor whose values are tabulated depending on the ratio between the maximum and the minimum moment of inertia of the column [9]. Lee et al proposed calculates the buckling load of the tapered column via the one of a uniform column of minimum section using a modification factor [10]. The same idea was studied by Hirt and Crisinel in [11] but the uniform is of equivalent section whose the moment of inertia is in practical form. Marques et al [12] proposed an analytic solution for the buckling load of the tapered column. However, in our knowledge, the buckling of tapered steel column with flexible beam-column joint is still less studied.

This paper performs the reliability assessment of buckling Strength for Battened built-up Columns steel considering shear deformations. An algorithm using Monte Carlo simulation method is proposed to utilize in analyses and assessments. In addition, effects of input parameters, which are safety factor, on reliability of structures are examined in the present study.

2. Theoretical framework

2.1. The effect of shear deformations on the elastic critical column load

In this section, presented the effect of shear deformations on the elastic critical column load, the differential equation, that presents the equilibrium of the column, has the form [1, 13].

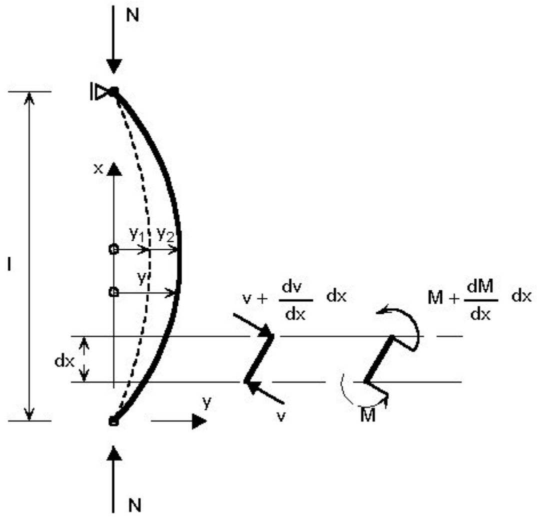

Figure 1. Pin-ended column

Considering the simple case of a pin-ended column as shown in Figure 2, where, M, N, V, x and y as defined in this Figure 1. The equilibrium conditions

$$M = N.y; \quad V = \frac{dM}{dx} = N\frac{dy}{dx} \tag{1}$$

The total lateral deflection *y* of the centerline is the result of two components: The bending moment *M* gives rise to the deflection y_1, and the shearing force *V* to the additional deflection y_2.

$$y = y_1 + y_2 \tag{2}$$

According to elastic theory the curvature due to the bending moment *M* is as follows:

$$\frac{d^2 y_1}{dx^2} = -\frac{M}{EI} = -\frac{Ny}{EI} \tag{3}$$

where, *E* is the modulus of elasticity (Young's modulus); *I* is the moment of inertia of the cross-section. The slope due to the shearing force *V* is as follows:

$$\frac{dy_2}{dx} = \beta\frac{V}{GA} = \beta\frac{N}{GA}\frac{dy}{dx} \tag{4}$$

where, *A* is the cross-sectional area; *G* is the modulus of rigidity or shear modulus; β is the shape factor of the column cross-section (β =1,11 for solid circular cross-sections; β = 1,2 for rectangular cross-sections). The curvature due to the effect of the shearing force *V* is as follows:

$$\frac{d^2 y_2}{dx^2} = \beta\frac{1}{GA}\frac{dV}{dx} = \beta\frac{N}{GA}\frac{d^2 y}{dx^2} \tag{5}$$

The total curvature of the buckling curve is due both to the bending moment, Eq.(3), and to the shearing force, Eq.(5).

$$\frac{d^2y}{dx^2} = \frac{d^2y_1}{dx^2} + \frac{d^2y_2}{dx^2} = -\frac{Ny}{EI} + \beta\frac{N}{GA}\frac{d^2y}{dx^2} \quad (6)$$

It is possible to rearrange Eq.(6) in the form

$$\frac{d^2y}{dx^2} + \frac{N}{\left(1-\frac{\beta N}{GA}\right)EI}y = 0 \quad (7)$$

Adopting the same procedure as in the Euler case, the critical load is defined by the equation:

$$\frac{N}{\left(1-\frac{\beta N}{GA}\right)EI}y = \frac{\pi^2}{l^2} \quad (8)$$

Solving for N, the following expression for the elastic critical load $N_{cr,id}$ is obtained:

$$N_{cr,id} = \frac{1}{\left(\frac{1}{N_{cr}} + \frac{1}{S_v}\right)} = N_{cr}\frac{1}{1+\frac{N_{cr}}{S_v}} \quad (9)$$

Where, $N_{cr} = \pi^2 EI / l^2$ is the Euler buckling load obtained disregarding the deformations due to shearing force; S_v is the shear stiffness of the column.

2.2. Evaluation of the shear stiffness of Battened columns

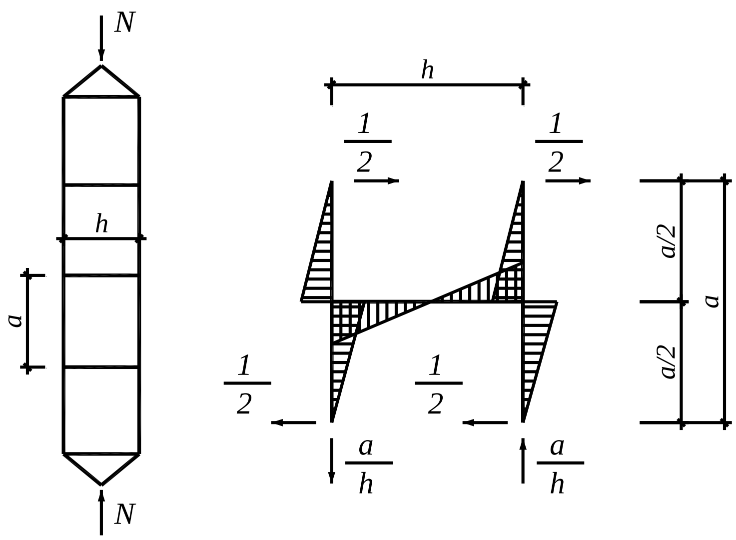

Figure 2. Battened built-up columns

Considering battened built-up columns, as shown in Figure 2. According [1, 13] adopting the virtual work method, the displacement d due to unit shearing force is obtained shown as

$$\delta = 4\int_0^{a/2}\frac{1}{2}\frac{x}{EI_c}\frac{1}{2}xdx + 2\int_0^{h/2}\frac{a}{h}\frac{y}{EI_b}\frac{a}{h}ydy = \frac{a^3}{24EI_c} + \frac{a^2h}{12EI_b} \quad (10)$$

The shear stiffness be determined

$$\frac{1}{S_v} = \frac{a^2}{24EI_c} + \frac{ah}{12EI_b} \tag{11}$$

Where, I_c is the in-plane second moment of area of one chord; I_b is the in-plane second moment of area of one batten.

2.3. Monte Carlo simulation

Monte Carlo simulation method is based on the use of pseudo-random numbers and the law of large number to assess the reliability of any system. If the safe domain is defined by the condition $f(\boldsymbol{X}) > 0$, where $\boldsymbol{X}$ is a random vector containing all the input random variables, the unsafe probability of the system is determined by:

$$P_f = \int I_{f(X)<0} f_X(x)dx = E\left[I_{f(X)<0}\right] \tag{12}$$

where, $I_{f(X)<0}$ is the indicator function and is defined by.

$$I_{f(X)<0} = \begin{cases} 1 & \text{if} \quad f(X) < 0 \\ 0 & \text{if} \quad f(X) \geq 0 \end{cases} \tag{13}$$

According to the theory of statistics, if we have N realizations of the random vector $\boldsymbol{X}$, by propagating the randomness, we obtain a sample of N realizations of the indicator function. The expected value of the indicator function can be approximately determined by taking the mean of the sample.

$$\widehat{P}_f = E\left[I_{f(X)<0}\right] = \frac{1}{N}\sum_{i=1}^{N} I^i_{f(X)<0} \tag{14}$$

A 95% confidence interval of the estimation is defined by [14]

$$\widehat{P}_f\left(1-200\sqrt{\frac{1-\widehat{P}_f}{N\widehat{P}_f}}\right) \leq P_f \leq \widehat{P}_f\left(1+200\sqrt{\frac{1-\widehat{P}_f}{N\widehat{P}_f}}\right) \tag{15}$$

The reliability assessment by Monte Carlo simulation is constructed in MATLAB language.

3. Methodology

3.1. Safe Condition

According to Eq.(9) from sections 2.1 the safe state of the elastic critical column load considering shear deformations, but in reality we often introduce a safety factor n and the safe condition will be written as following.

$$N \leq \frac{N_{cr,id}}{n} \tag{16}$$

Where, N is the external load that can fluctuate randomly in the exploration of the structures.

3.2. Deterministic Models and Uncertainty Models

The deterministic model deals with the safe state of the elastic critical column load considering shear deformations, in which the input parameters are those of geometry (L, HEB, A_w), material properties (E, υ), loading (N) is model can be written shown as Eq.(15) with $\boldsymbol{X} = [L, HEB, a, h, I_b, I_c, N, E]$.

$$N_{saf} = \Im(\mathbf{X}) \tag{17}$$

The uncertainty model is constructed based on the deterministic model by taking into account the randomness of some input parameters. In this paper, we distinct two vectors of input parameters: the first one of the parameters assumed to be deterministic $\boldsymbol{X}_1 = [L, \upsilon]$ and the second one of the parameters assumed to be random $\boldsymbol{X}_2(\omega) = [HEB(\omega), a(\omega), h(\omega), I_b(\omega), I_c(\omega), N(\omega), E(\omega)]$ with ω representing the randomness of the parameters. This model can be written as:

$$N_{saf} = \Im\left(\boldsymbol{X_1}, \boldsymbol{X_2}(\omega)\right) \tag{18}$$

3.3. Reliability assessment of Buckling Strength for Battened built-up Columns steel considering shear deformations

By introducing the uncertainty model in the Monte Carlo simulation method, we obtain the scheme of the reliability assessment of the elastic critical column load considering shear deformations as shown in Figure 3.

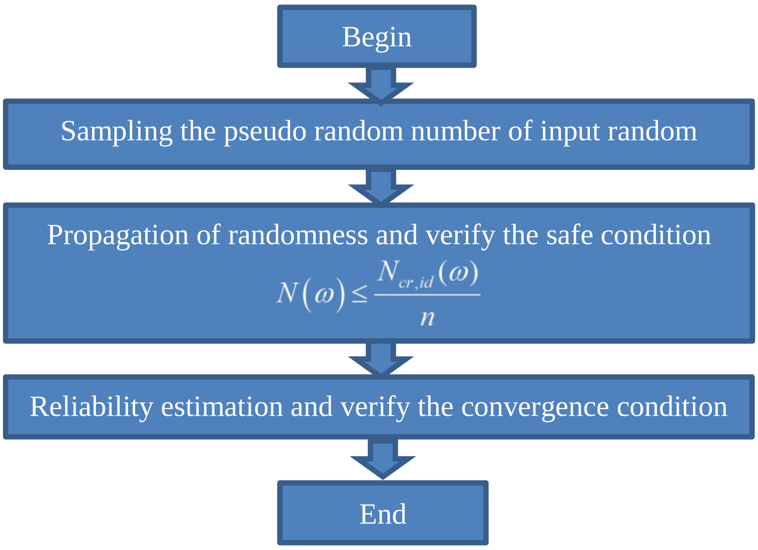

Figure 3. Flowchart of the reliability assessment of the Buckling strength considering shear deformations by Monte Carlo simulation method

4. Numerical results

4.1. Effect of shear on the elastic critical column load of Battened column

In the section, the study will be presented effect of shear on the elastic critical column load of Battened column steel. Consider the Battened column steel as shown in Figure 4 and input parameter shown in Table 1. The calculated results as shown in Table 2 obviously $N_{cr,id} < N_{cr}$. This means, considering the effect of shear on the elastic critical column load of Battened column steel is necessary.

Table 1. Input parameter for Battened column

Properties	Variables	Value	Unit
Material	E	200,0	(N/mm^2)
	υ	0,30	-
Cross-section	a	200,0	cm
	h	100,0	cm
	I_c	8564,0	cm^4
	I_b	21333,0	cm^4
Geometry	L	1000,0	cm

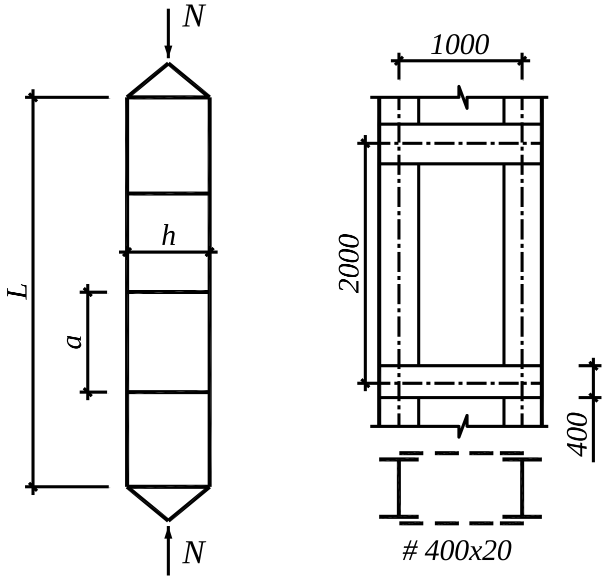

Figure 4. Battened built-up columns

Table 2. The results of N_{cr}, S_v and $N_{cr,id}$

N_{cr} (MN)	S_v (MN)	$N_{cr,id}$ (MN)
16,90	73,33	13,74

4.2. Convergence of the Monte Carlo simulation

Consider the steel columns as shown in Figure 4 with the deterministic input parameters and the random input parameters variable shown in Table 3. The nominal and distribution of material properties on based proposed in [15]. The cross-section and loading on based proposed in [16].

Table 3. Statistical properties of random variables for Built-up column steel

Properties	Variables	Nominal	Mean/ nominal	COV	Distribution	Reference
Material	E	200 (N/mm^2)	1,10	0,06	Lognormal	[15]
	υ	0,30	-	-	-	-
Cross-section	a	200 (cm)	1,00	0,05	Normal	[16]
	h	100 (cm)	1,00	0,05	Normal	[16]
	I_c	8564,0 (cm^4)	1,00	0,05	Normal	[16]
	I_b	21333,0 (cm^4)	1,00	0,05	Normal	[16]
Geometry	L	1000,0 (cm)	-	-	-	-
Loading	N	14,70 (kN)	1.05	0.10	Normal	[16]

Figure 5 shows the convergence of the safe probability of the column in the Monte Carlo simulation to the value of 0,9494 or 94,94% after about 1750 sampling in 11 minutes. The used convergence criteria of 2,5% justifies the confidence of the estimated reliability. This result also shows that although we have taken the safety factor is 1,15 in the analysis but because of the randomness of some input parameters, the reliability of the structure is only of 94,94%. The assessment of the reliability of the structure thus is necessary.

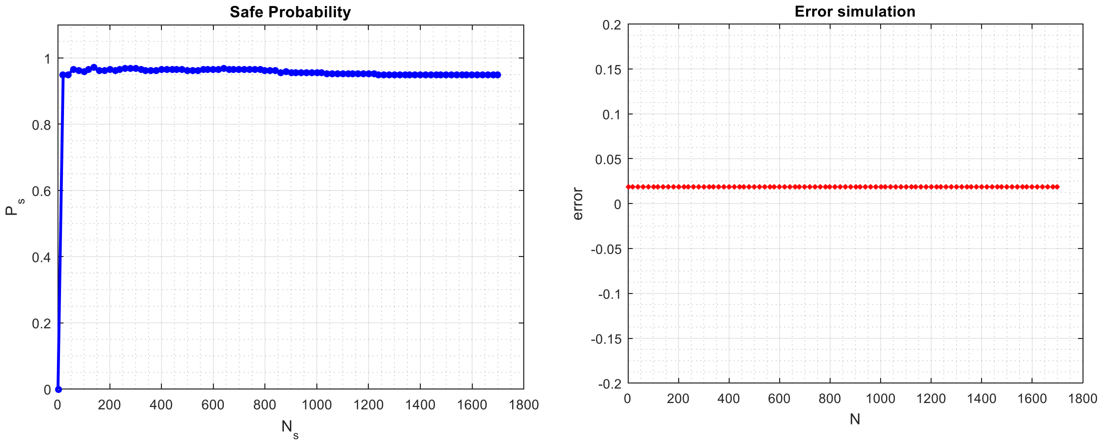

Figure 5. Convergence of the safe probability in the Monte Carlo simulation

4.3. Effect of the coefficient safety

We know clearly that the variation of input random variables and the safety factor influence directly but inversely on the safe probability of the structure. Thus in order to clear the effect of these parameters, one reconsiders the above column with different coefficients of safety $n = 1{,}15;\ 1{,}2;\ 1{,}25;\ 1{,}30;\ 1{,}35$. The results of the safe probability are listed in Table 4 and presented in Figure 6.

Table 4. Effect of coefficients of safety on the safe probability

n	1,15	1,2	1,25	1,30	1,35
$P_s(\%)$	94,94	97,42	98,53	99,37	100,00

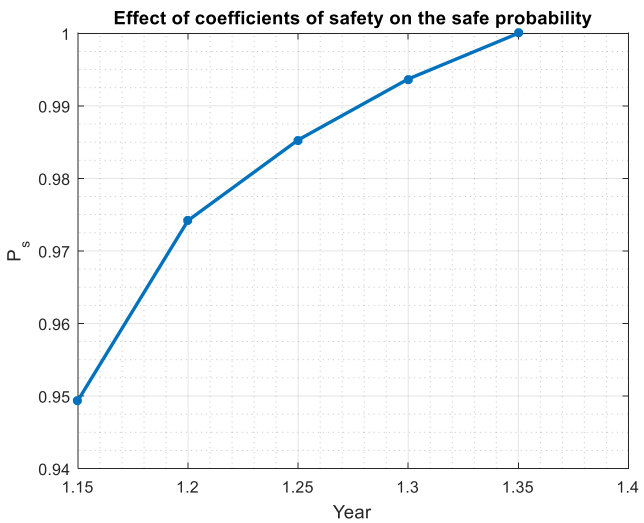

Figure 6. Effect of coefficients of safety on the safe probability

We can easily observe the effect of inverse of the safety factor in Table 4. The safe probability of the column increases when the safety factors increase. Safety factors help designers adjust the desired reliability.

5. Conclusions

This paper proposed an algorithm to assess the structural reliability of buckling Strength for Battened built-up Columns steel considering shear deformations. The numerical process is developed based on the Monte Carlo simulation. The effect of safe factor on probability of safety is also examined. The flowing conclusions are drawn based on numerical analyses.

- The proposed algorithm, which is numerically developed based on the Monte Carlo simulation, is capable of structural reliability assessment of buckling Strength for Battened built-up Columns steel considering shear deformations.
- The probability of safety is increased together with an increment of state factor.

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