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AN ANALYSIS OF THE STRESS-STRAIN STATE OF REINFORCED CONCRETE BEAM CROSS-SECTIONS SUBJECTED TO BIAXIAL BENDING

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ABSTRACT: This paper presents a simple analytical method for determining the stress-strain state of reinforced concrete (RC) beams subjected to biaxial bending. A simple supported beam with three input load values applied at two positions is used in this study. The beam cross-section is divided into numerous elements and then the stress-strain state at the centre of these elements is analysed. The material behaviour of concrete and rebar are both simulated using a bilinear model. The results of this method are compared and verified with a three-dimensional finite element approach. The results indicate that the proposed method is easy to implement and its performance is comparable to a numerical simulation method. The stress-strain state, location, and shape of the neutral axis, as well as the shape of the compressive and tensile area can be extracted using the proposed method. This method can also present the position and length of the crack in the cross-section. Moreover, this method ensures accurate evaluation when selecting materials and the initial layout of the rebars. The testing and evaluation of the accuracy of the material selection can be completed by using this proposed method. Also, the method can be applied to RC beams with different cross-sections and complex loading.

Keywords: Analytical method, Stress-strain state, Biaxial moment, RC beam.

1. INTRODUCTION

The problem of analysing the load in the bending plane of the stress-strain state of flexural reinforced concrete beams has been solved by many different methods. However, where the load is not in the bending plane (biaxial bending), such as with purlins and outside beams, the stress-strain state is complex. Thus, few methods are available to analyse the stress-strain state of the cross-section of reinforced concrete beams with biaxial bending. To analyse this problem, it is possible to use the finite element method, particularly with the aid of simulation software, such as SAP, ETABS, ABAQUS, or ANSYS. However, the finite element method has several shortcomings. For example, computational analysis is difficult and resource-intensive when dividing too many elements, if the element has many nodes.

Several studies have examined the biaxial bending problem of reinforced concrete (RC) structures. However, the results to date lack accuracy because they use closed solutions so as to create simpler problems.

Dundar [1] proposed a procedure for calculating the cross-section of a box shape and used the Newton-Raphson method to solve the stress-strain problem of the biaxial bending section with axial load; a calculation program with many loops was run until the problems converged and found the approximate position of the neutral axis of the cross-section. Yau, Chan and So [2] proposed a procedure for designing RC cross-sections with biaxial bending based on the position of the neutral axis and the percentage of reinforcement area (ρ). Many assumptions were made for simplicity in the model, and these greatly affect the accuracy of the results. Yen et al [3] proposed a procedure for determining the strength of sections under biaxial bending using the parameter, ρ, which is the distance between the neutral axis of the section and the point with maximum compressive stress. Zak [4] proposed a procedure for calculating the strength of the cross-section of structural members subject to biaxial bending using the Newton-Raphson method, using many examples to analyse the results. Hsu C. [5] conducted experiments with numerous experimental T-beam samples under different types of load, including biaxial bending, and proposed a method for calculating the strain-stress state of the T-beam. Charalampakis [6] presented algorithms for analysing the stress state of a cross-section subject to biaxial bending and an axial force.

In these studies, many approximations were used, as complex methods make application difficult and affect the reliability of the results. Most of the earlier proposals are used in design with approximate assumptions, but there is still difficulty with these methods in problem-solving. Thus, this study describes a method for calculating the stress-strain state of a section of RC under biaxial bending. The method has the advantages of accuracy, simple calculation, and ease of use.

2. RESEARCH SIGNIFICANCE

The stress-strain state analysis of flexural reinforced concrete beams with applied load in the bending plane has been solved by many different methods. However, for biaxial bending (the load is not in the bending plane), the problem becomes complex, and few methods are available, both numerical and analytical. The numerical has several inadequacies such as time-consuming and costly. On the other hand, the analytical has a lot of approximate assumptions.

This study presents a method for determining the stress-strain state of a cross-section of reinforced concrete (RC) beams subjected to biaxial bending. This method is easy to implement with acceptable accuracy. Accordingly, a simple supported beam with three input load values applied at two positions is used in this research. The beam cross-section is divided into numerous elements, and then the stress-strain state at the center of these elements is analyzed. The material behavior of concrete and rebar are both simulated using a bilinear model. The results of this method are compared and verified with the finite element method (ANSYS software).

By using this proposed method, the location and shape of the neutral axis, as well as the shape of the compressive and tensile area of the RC cross-section can be determined. Moreover, the shape, position, and length of the crack in the cross-section can be observed. Furthermore, this method ensures accurate evaluation when selecting materials and the initial layout of the rebars. Also, the proposed analytical method can also be used for different cases, such as various values of load and cross-sectional shapes.

3. CONTENTS

Two methods of analysing the stress-strain state of RC beams with biaxial bending are performed and compared. The first method is the stress-strain analysis of the cross-section at the centre of the beam (Fig. 1). The cross-section is divided into many elements and then the stress-strain of the element at its centre is analysed. With this method, it is assumed that in the microscopic elements, the positions around the centre element work in the same way as other elements. The second method is to use a simulation with ANSYS software, which uses a finite element basis for analysis. The results of these two methods are analysed and compared.

3.1 Proposed Analytical Method

The method of stress-strain analysis presented in this research divides a cross-section into many elements and then performs stress-strain analysis on the centre of those elements. This is the main way in which the complexity of the computation is reduced.

An RC beam with biaxial bending was chosen to analyse the stress-deformation state of the section (Fig. 1). The model uses an RC beam with a joint of the two ends, and the load P_0 is placed at two positions on the beam (Fig. 1). The point P_0 deviates from the bending plane, and the direction of P_0 from the vertical is angle α. The problem is to conduct the stress-strain state analysis of the rectangular cross-section of the RC beam that has the greatest stress with biaxial bending. Fig. 2 shows the detail of the section, layout of the reinforcing bar, and reinforcement distance.

Concrete materials C12/15 were selected according to the Eurocode standard [7], reinforced with Grade 250 cast iron [8]. Physical characteristics are according to the Eurocode standard, as follows:

Concrete C12/15 has calculated strength, compressive strength $f_{cd} = 8MPa$, tensile strength $f_{ctd} = 0.73MPa$, $\varepsilon_{c3} = 0.00175$; $\varepsilon_{cu3} = 0.0035$; The values of the tensile concrete design strain can be ignored, but this study still considers the bilinear hypothesis for the compression zone, according to [9], as follows:

$$\varepsilon_{ct3} = 8 \cdot 10^{-5}; \tag{1}$$

$$\varepsilon_{ctu3} = 15 \cdot 10^{-5}; \tag{2}$$

Where:

ε_{c3}, ε_{ct3}: the largest strain corresponds to the stage of elastic compression and tension;

ε_{cu3}, ε_{ctu3}: the largest strain of the compressive and tensile concrete [7];

$E_{c,red}$, $E_{ct,red}$: converted modulus of compressive, tensile concrete;

The Grade 250 iron [8] has the following parameters:

$$f_{pd} = 250MPa; \tag{3}$$

$$E = 2 \cdot 10^5 MPa; \tag{4}$$

$$\varepsilon_{s3} = \frac{f_{cd}}{E_s} = \frac{250}{2 \cdot 10^5} = 1.25 \cdot 10^{-3}; \tag{5}$$

$$\varepsilon_{su3} = 25 \cdot 10^{-3}. \tag{6}$$

The elastic modulus of reinforcement is: $E_s = 2 \cdot 10^5 MPa$. (7)

The bilinear deformation model of concrete is shown in Fig. 3 and the reinforcement is shown in Fig. 4.

The stress-strain curve of the bilinear model of concrete C12/15 are set out below. For the compression area:

- $-175\cdot10^{-5} < \varepsilon_{ci} < 0$:

$$\sigma_{bi} = \varepsilon_{bi}\cdot E_{b,red} \text{ MPa;} \tag{8}$$

$$E'_{ci} = E_{c,red} = \frac{f_{cd}}{175\cdot10^{-5}} = 4571 \text{ MPa.} \tag{9}$$

- $-350\cdot10^{-5} < \varepsilon_{ci} < -175\cdot10^{-5}$:

$$\sigma_{ci} = f_{cd} = 8 \text{ MPa;} \tag{10}$$

$$E'_{ci} = \frac{f_{cd}}{\varepsilon_{ci}} = \frac{8}{\varepsilon_{ci}} \text{MPa.} \tag{11}$$

- $\varepsilon_{ci} < -350\cdot10^{-5}$:

$$\sigma_{ci} = 0; \tag{12}$$

$$E'_{ci} = 0. \tag{13}$$

For the tension area:

- $0 < \varepsilon_{cti} < 8\cdot10^{-5}$:

$$\sigma_{cti} = \varepsilon_{cti}\cdot E_{ct,red} \text{ MPa;} \tag{14}$$

$$E'_{cti} = E_{ct,red} = \frac{f_{ctd}}{\varepsilon_{ct3}} = 9125 \text{ MPa.} \tag{15}$$

- $8\cdot10^{-5} < \varepsilon_{cti} < 15\cdot10^{-5}$:

$$\sigma_{cti} = f_{ctd} = 0.73 \text{ MPa;} \tag{16}$$

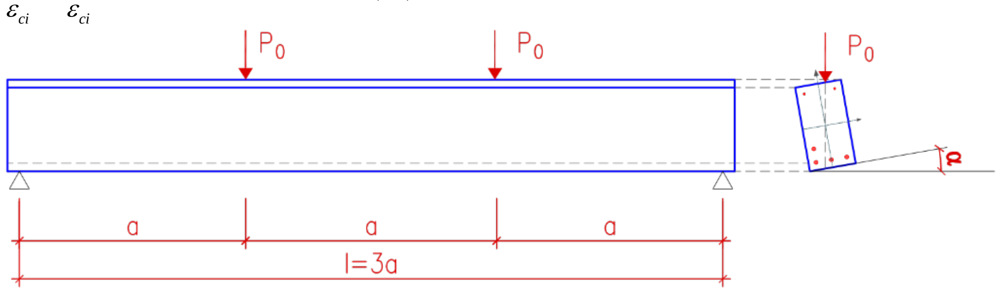


Fig. 1. Model of the reinforced concrete beam for biaxial bending

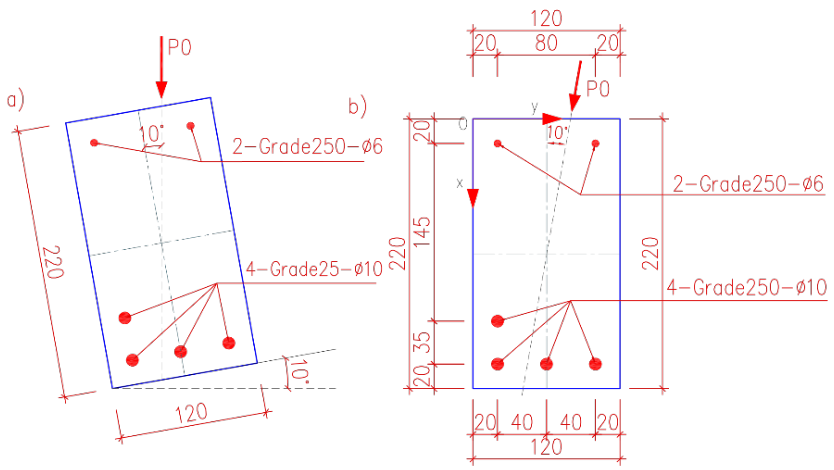


Fig. 2. Cross-section of the reinforced concrete beam, a) rebar layout, b) dimensions in detail

Fig. 1 and Fig. 2 describe the detail of RC beams subjected to biaxial bending. The cross-section was rectangular with 120 mm (width) x 220 mm (height) and inclined 10 degrees to the vertical. Only longitudinal steel reinforcement was placed, no stirrups, due to the beam considers to flexural.

Two 6 mm diameter bars were set at the top of the beams, while four 10 mm diameter bars were set at the bottom. The concrete grade was 250 with a cover of 20 mm in the four faces. The distance of bottom rebars was 40 mm in horizontal and 35 mm in vertical.

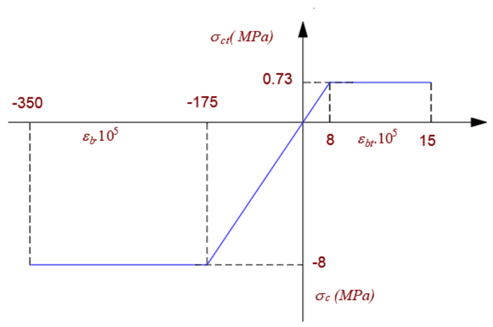


Fig. 3. Bilinear stress-strain relationship of concrete

$$E_{cti}^{'} = \frac{0.73}{\varepsilon_{cti}} \text{ MPa.} \tag{17}$$

- $15 \cdot 10^{-5} < \varepsilon_{cti}$:

$$\sigma_{bti} = 0; \tag{18}$$

$$E_{cti}^{'} = 0. \tag{19}$$

To use the bilinear strain-stress relationship, we accept the following assumptions:

- Small elements of the cross-section are considered to work homogeneously, that is, the strain and stress in each element of the cross-section are the same.
- Flat cross-sections are used for the bending beams [10].
- Under the impact of load, beams are bent in a certain direction with a certain radius of curvature.

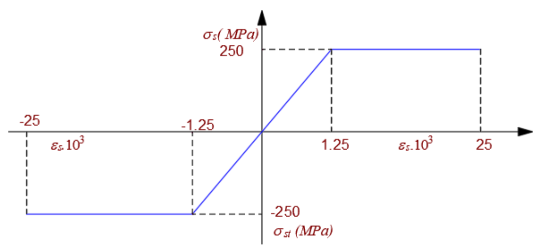


Fig. 4. The bilinear stress-strain relationship of the reinforcement

With these concepts for the calculation, we divide the cross-section into many small parts as shown in Fig. 5. I and j are the indices of the elements for the x and y-axis respectively (for the selected coordinates O_{xy}). The x-axis is divided into *i* parts, the y-axis into *j* parts.

- $Z_{cxij}; Z_{cyij}$ are the distances from the centre of the concrete elements to the y and x-axis;
- $Z_{sxij}; Z_{syij}$ are the distances from the centre of the reinforcement elements to the y and x-axis;
- M_x, M_y are the moments of the beam about the y and x-axis.

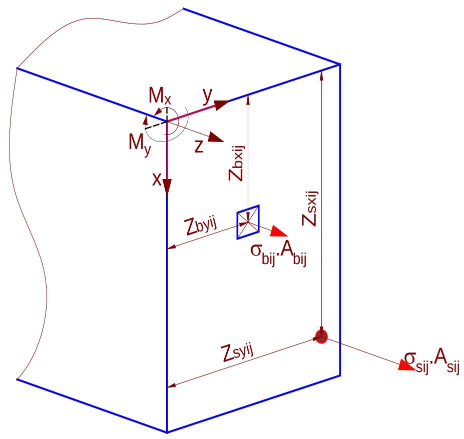


Fig. 5. The cross-section shows the parameters of concrete and reinforcement elements.

This analytical method is based on the stress-strain relationship of the concrete and the reinforcing material. The calculation process is made up of many steps; the first step takes the value of the elastic modulus. In the next step, the modulus value is taken from the stress and strain according to the following formulae:

$$E_{cij} = \frac{\sigma_{cij}}{\varepsilon_{bij}} \tag{20}$$

$$E_{sij} = \frac{\sigma_{sij}}{\varepsilon_{sij}} \tag{21}$$

With $E_{cij}; E_{sij}$ are the element modulus i, j of the concrete, and the reinforcement, respectively.

The formulae determine the internal force values according to [9]:

$$M_x = D_{11} \cdot \frac{1}{r_x} + D_{12} \cdot \frac{1}{r_y} + D_{13} \cdot \varepsilon_0 \tag{22}$$

$$M_y = D_{21} \cdot \frac{1}{r_x} + D_{22} \cdot \frac{1}{r_y} + D_{23} \cdot \varepsilon_0 \tag{23}$$

$$N = D_{31} \cdot \frac{1}{r_x} + D_{32} \cdot \frac{1}{r_y} + D_{33} \cdot \varepsilon_0 \tag{24}$$

Where: ε_0 is the strain of the selected origin of the coordinate's axis O_{xy}, the coefficients $D_{m,n}; (m, n = 1, 2, 3)$ Equations (41, 42, 43) can be rewritten:

$$D_{11} = \sum_i \sum_j A_{cij} \cdot Z_{cxij}^2 \cdot E_{cij} + \sum_i \sum_j A_{sij} \cdot Z_{sxij}^2 \cdot E_{sij} \quad (25)$$

$$D_{22} = \sum_i \sum_j A_{cij} \cdot Z_{cyij}^2 \cdot E_{cij} + \sum_i \sum_j A_{sij} \cdot Z_{syij}^2 \cdot E_{sij} \quad (26)$$

$$D_{12} = \sum_i \sum_j A_{cij} \cdot Z_{cxij} \cdot Z_{cyij} \cdot E_{cij} + \sum_i \sum_j A_{sij} \cdot Z_{sxij} \cdot Z_{syij} \cdot E_{sij} \quad (27)$$

$$D_{13} = \sum_i \sum_j A_{cij} \cdot Z_{cxij} \cdot E_{cij} + \sum_i \sum_j A_{sij} \cdot Z_{sxij} \cdot E_{sij} \quad (28)$$

$$D_{23} = \sum_i \sum_j A_{cij} \cdot Z_{cyij} \cdot E_{cij} + \sum_i \sum_j A_{sij} \cdot Z_{syij} \cdot E_{sij} \quad (29)$$

$$D_{33} = \sum_i \sum_j A_{cij} \cdot E_{cij} + \sum_i \sum_j A_{sij} \cdot E_{sij} \quad (30)$$

The strain of each concrete and reinforcement element is determined by the following formulae:

$$\varepsilon_{bij} = \frac{1}{r_x} \cdot Z_{cxij} + \frac{1}{r_y} \cdot Z_{cyij} + \varepsilon_0 \quad (31)$$

$$\varepsilon_{sij} = \frac{1}{r_x} \cdot Z_{sxij} + \frac{1}{r_y} \cdot Z_{syij} + \varepsilon_0 \quad (32)$$

In the first calculation, we used elastic modulus values $E_{cij}; E_{sij}$ in equations (20) to (21) as follows:

For concrete: $E_{cij} = E_{c,red}$,[9];

For reinforcement: $E_{sij} = E_s$.

In the next calculation steps, the calculation method is repeated but the module value is obtained according to formulae (31) and (32). The result of the problem is when there is no longer a deviation in the centre curvature of the elements in the flex plane. Thus, the problem will have many calculations within the application; the results of the calculation process are accepted when the curvature is less than 1%. The stress-strain of that step is accepted as the result and the calculation of the stress-strain state of the structural section is completed. When calculating with the beam subjected to oblique bending as above, the process of subdivision cross-section is as shown in Fig. 6. The selected coordinate system and the centre coordinate of the elements are shown in Fig. 7.

The origin O is selected at the left angle of the section, the x-axis is the downward vertical, and the y-axis is horizontal. With a rectangular cross-section, it is straightforward to determine the parameters for the size, area, and centre coordinates of the elements.. The x-axis is divided into 12 parts, and the y-axis 7 parts. For simplicity, we used matrix operations to process the requirements, programmed in MathCad 15. The element positions and their characteristics are made up of a matrix of 12 rows and 7 columns. Equations (20) through (32) are performed on matrix calculations. The following presents the results of the above method with the support of MathCad 15 software.

The analysis was performed using three torque values:

$M_1 = 10^6\, N.mm$; $M_2 = 5 \cdot 10^6\, N.mm$; $M_3 = 10 \cdot 10^6\, N.mm$ (with the assumption that the inclination angle of the applied force P_0 is α = 100 relative to the vertical axis). The force along the beam axis $N = 0$.

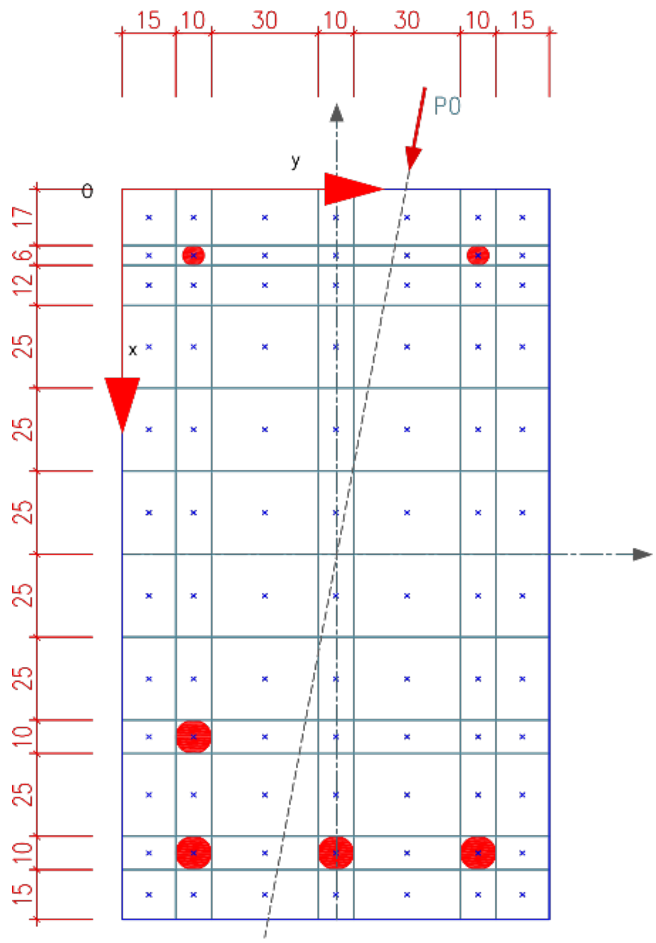


Fig. 6. Mesh cross-section

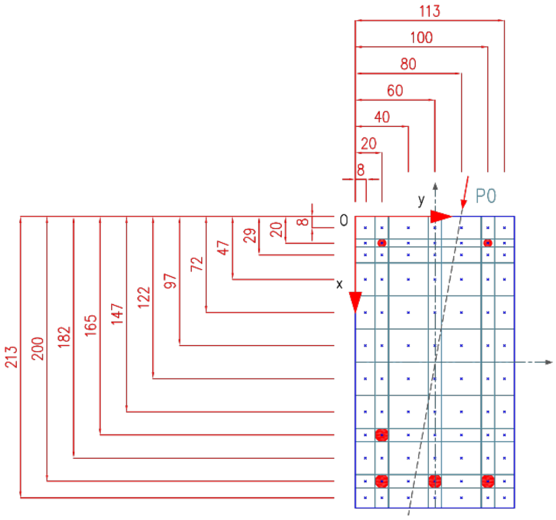


Fig. 7. The coordinates of the elements

3.2 Analysis by ANSYS

3.2.1 The theoretical basis of concrete

The stress-strain relationship of the concrete was determined using the method from the study of Vecchio and Collins [11]. With concrete strength $f = 8$ MPa, the stress-strain relationship curve is shown in Fig. 8. The maximum strain value of concrete $\varepsilon_{\max} = 0.0035$ is taken according to the Euro Code [12], and the corresponding elastic modulus identified by Vecchio and Collins [11] is $E_c = 7280$ MPa. The strain at the top of the stress-strain relation curve is $\varepsilon_0 = 0.002$. In this case, concrete is selected as the SOLID65 element [13].

3.2.2 The theoretical basis of reinforcement

We propose the stress-strain relationship of the reinforcement using the bilinear relation as shown in [14], the stress-strain relationship of the reinforcement with two necessary parameters, reinforcement elastic modulus (E_s), and yield strength (f_y) (Fig.9).

The analytical model was established with the help of graphics software Design Modeler; the results of the analytical model are shown in Fig. 10. Due to the symmetrical structure of the beam, only half the structure is considered in order to reduce the analytical volume of the process. After modelling, analysis was carried out with Mechanical (ANSYS Static), then the properties of the concrete and reinforcement were added as code snippets corresponding to the models. To show the simultaneous working between concrete elements and reinforcing elements, ANSYS software allows simulation of the relative displacement between SOLID65 and LINK180 element types in the form of CEINTF commands.

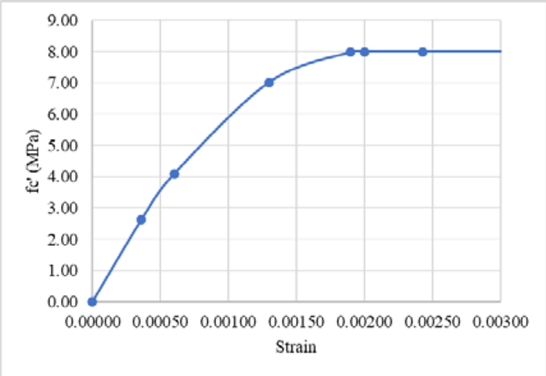


Fig. 8. The relationship of stress - deformation state of concrete

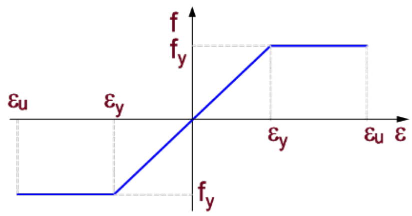


Fig. 9. The stress-strain relationship of reinforcement

4. RESULTS

The stress-strain state of the cross-section of the RC beam was analysed for three different load values. Results for the two analytical methods are shown below.

4.1 Recommended Analysis Method

4.1.1 Case 1, with assumed torque $M_1 = 10^6\, N.mm$

Table 1. The deviation between calculations steps for case 2

Times	Results			Declination (%)		
	r_x	r_y	ε_0	r_x	r_y	ε_0
1	932000	-1350000	-0.0000983	-	-	-
2	1012000	-1686000	-0.0001014	-8.58	-24.89	-3.17
3	989200	-1557000	-0.0000994	2.25	7.65	1.94
4	984300	-1537000	-0.0000993	0.50	1.28	0.12

Table 2. The strain at the centre of concrete elements at the cross-section between beams

		Column						
		1	2	3	4	5	6	7
Row	1	-9.71E-05	-1.05E-04	-1.19E-04	-1.32E-04	-1.46E-04	-1.59E-04	-1.68E-04
	2	-8.46E-05	-9.27E-05	-1.06E-04	-1.20E-04	-1.33E-04	-1.47E-04	-1.56E-04
	3	-7.52E-05	-8.33E-05	-9.68E-05	-1.10E-04	-1.24E-04	-1.37E-04	-1.46E-04
	4	-5.64E-05	-6.45E-05	-7.81E-05	-9.16E-05	-1.05E-04	-1.19E-04	-1.27E-04
	5	-3.04E-05	-3.85E-05	-5.20E-05	-6.55E-05	-7.90E-05	-9.25E-05	-1.01E-04
	6	-4.33E-06	-1.24E-05	-2.60E-05	-3.95E-05	-5.30E-05	-6.65E-05	-7.53E-05
	7	2.17E-05	1.36E-05	9.69E-08	-1.34E-05	-2.69E-05	-4.04E-05	-4.92E-05
	8	4.78E-05	3.97E-05	2.62E-05	1.26E-05	-8.76E-07	-1.44E-05	-2.32E-05
	9	6.65E-05	5.84E-05	4.49E-05	3.14E-05	1.79E-05	4.37E-06	-4.42E-06
	10	8.42E-05	7.61E-05	6.26E-05	4.91E-05	3.56E-05	2.21E-05	1.33E-05
	11	1.03E-04	9.49E-05	8.14E-05	6.79E-05	5.44E-05	4.08E-05	3.21E-05
	12	1.17E-04	1.08E-04	9.49E-05	8.14E-05	6.79E-05	5.44E-05	4.56E-05

Table 3 Strain at the centre of reinforcing elements at the cross-section between beams

		Column						
		1	2	3	4	5	6	7
Row	1	0	0	0	0	0	0	0
	2	0	-0.0000927	0	0	0	-0.0000302	0
	3	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0
	8	0	0	0	0	0	0	0
	9	0	0.0000584	0	0	0	0	0
	10	0	0	0	0	0	0	0
	11	0	0.0000949	0	0.0000679	0	0.0000408	0
	12	0	0	0	0	0	0	0

Table 4. Stress value at the centre of concrete elements at the cross-section between of beams, Mpa

		Column						
		1	2	3	4	5	6	7
Row	1	-0.44	-0.48	-0.54	-0.60	-0.67	-0.73	-0.77
	2	-0.39	-0.42	-0.49	-0.55	-0.61	-0.67	-0.71
	3	-0.34	-0.38	-0.44	-0.50	-0.57	-0.63	-0.67
	4	-0.26	-0.30	-0.36	-0.42	-0.48	-0.54	-0.58
	5	-0.14	-0.18	-0.24	-0.30	-0.36	-0.42	-0.46
	6	-0.02	-0.06	-0.12	-0.18	-0.24	-0.30	-0.34
	7	0.10	0.06	0.00	-0.06	-0.12	-0.19	-0.23
	8	0.22	0.18	0.12	0.06	0.00	-0.07	-0.11
	9	0.30	0.27	0.21	0.14	0.08	0.02	-0.02
	10	0.73	0.35	0.29	0.22	0.16	0.10	0.06
	11	0.73	0.73	0.73	0.31	0.25	0.19	0.15
	12	0.73	0.73	0.73	0.73	0.31	0.25	0.21

Results for the cross-section at the centre of the beams obtained from MathCad 15 at each centre position of the concrete and reinforced elements are shown in Tables 2,3,4, and 5. Errors of the curvature radius in the direction of the coordinate system (O_{xy}) and the strain of the grain at the intersection of the axes are shown in Table 1, in which: r_x, r_y are the radius of curvature of the beam, respectively in the x and y directions, and ε_0 is the relative strain of the grain at the intersection of the selected axes (point O). Results are considered to converge when the deviation of the radius and strain of the grain located at the intersection of the selected axes is less than 5%. The results of stress and deformation of the section are accepted at the 4th calculation.

4.1.2 Case 2, with assumed torque $M_2 = 5 \cdot 10^6\ N.mm$

Results for the cross-section at the centre of the beams obtained from MathCad 15 software, at each centre position of the concrete and reinforced elements are shown in Tables 7, 8, 9, and 10. Errors of the curvature radius in the directions of the coordinate system O_{xy} and the strain of the grain at the intersection of the axes are shown in Table 6, in which r_x, r_y are the radius of curvature of the beam, respectively in the x and y directions, and ε_0 is the relative strain of the grain at the intersection of the selected axes (point O). Results are considered converged when the deviation of the radius and relative deformation of the grain located at the intersection of the selected axes is less than 5%. The results of stress and deformation of the section are accepted at the 4th calculation.

Table 5. Stress value at the centre of reinforcing elements at the cross-section between of beams, Mpa

		Column						
		1	2	3	4	5	6	7
Row	1	0	0	0	0	0	0	0
	2	0	-18.5	0	0	0	-6.03	0
	3	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0
	8	0	0	0	0	0	0	0
	9	0	11.7	0	0	0	0	0
	10	0	0	0	0	0	0	0
	11	0	19	0	13.6	0	8.17	0
	12	0	0	0	0	0	0	0

Table 6. The deviation between calculations steps for case 2

Times	Results			Declination (%)		
	r_x	r_y	ε_0	r_x	r_y	ε_0
1	186400	-270100	-0.0004914	-	-	-
2	160900	-226400	-0.0005025	13.68	16.18	-2.26
3	158500	-210300	-0.0004740	1.49	7.11	5.67
4	158000	-215600	-0.0004851	0.32	-2.52	-2.34

The behavior results of the elements on the cross-section after analysis are made into a matrix of twelve rows and seven columns. The value at each matrix element corresponds to the value at the center of the cross-section element. The elements without reinforcement were set to 0. As shown in Table 6, the deviation of the curvature radius in the x-direction at calculation steps 1, 2, 3, and 4 were 13.68%, 1.49%, 0.32%, respectively. In this case, the analysis can be considered convergence at step 3. On the other hand, the deviation of the curvature radius in the y-direction x in calculation steps 1, 2, 3, and 4 were 16.18%, 7.11%, 2.52%, respectively. The analysis can be considered convergence at step 4. Overall, the analysis converges at step 4.

Table 7. Strain at the centre of concrete elements at the cross-section between beams

Row		Column						
		1	2	3	4	5	6	7
	1	-4.69E-04	-5.25E-04	-6.18E-04	-7.11E-04	-8.03E-04	-8.96E-04	-9.56E-04
	2	-3.93E-04	-4.49E-04	-5.42E-04	-6.35E-04	-7.27E-04	-8.20E-04	-8.80E-04
	3	-3.36E-04	-3.92E-04	-4.85E-04	-5.77E-04	-6.70E-04	-7.63E-04	-8.23E-04
	4	-2.22E-04	-2.78E-04	-3.71E-04	-4.63E-04	-5.56E-04	-6.49E-04	-7.09E-04
	5	-6.36E-05	-1.19E-04	-2.12E-04	-3.05E-04	-3.98E-04	-4.90E-04	-5.51E-04
	6	9.50E-05	3.93E-05	-5.34E-05	-1.46E-04	-2.39E-04	-3.32E-04	-3.92E-04
	7	2.54E-04	1.98E-04	1.05E-04	1.23E-05	-8.04E-05	-1.73E-04	-2.34E-04
	8	4.12E-04	3.56E-04	2.64E-04	1.71E-04	7.81E-05	-1.46E-05	-7.49E-05
	9	5.26E-04	4.71E-04	3.78E-04	2.85E-04	1.92E-04	9.95E-05	3.92E-05
	10	6.34E-04	5.78E-04	4.86E-04	3.93E-04	3.00E-04	2.07E-04	1.47E-04
	11	7.48E-04	6.92E-04	6.00E-04	5.07E-04	4.14E-04	3.21E-04	2.61E-04
	12	8.31E-04	7.75E-04	6.82E-04	5.89E-04	4.97E-04	4.04E-04	3.44E-04

Table 8 Strain at the centre of reinforcing elements at the cross-section between beams

Row		Column						
		1	2	3	4	5	6	7
	1	0	0	0	0	0	0	0
	2	0	-0.00045	0	0	0	-0.0000685	0
	3	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0
	8	0	0	0	0	0	0	0
	9	0	0.000471	0	0	0	0	0
	10	0	0	0	0	0	0	0
	11	0	0.000692	0	0.000507	0	0.000321	0
	12	0	0	0	0	0	0	0

Table 9. Stress value at the centre of concrete elements at the cross-section between of beams, MPa

Row		Column						
		1	2	3	4	5	6	7
	1	-2.15	-2.40	-2.82	-3.25	-3.67	-4.10	-4.37
	2	-1.80	-2.05	-2.48	-2.90	-3.32	-3.75	-4.02
	3	-1.54	-1.79	-2.22	-2.64	-3.06	-3.49	-3.76
	4	-1.02	-1.27	-1.69	-2.12	-2.54	-2.97	-3.24
	5	-0.29	-0.55	-0.97	-1.39	-1.82	-2.24	-2.52
	6	0.73	0.18	-0.24	-0.67	-1.09	-1.52	-1.79
	7	0.00	0.00	0.73	0.06	-0.37	-0.79	-1.07
	8	0.00	0.00	0.00	0.00	0.36	-0.07	-0.34
	9	0.00	0.00	0.00	0.00	0.00	0.73	0.18
	10	0.00	0.00	0.00	0.00	0.00	0.00	0.73
	11	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	12	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 10. Stress value at the centre of reinforcing elements at the cross-section between of beams, Mpa

		Column						
		1	2	3	4	5	6	7
Row	1	0	0	0	0	0	0	0
	2	0	-89.8	0	0	0	-13.7	0
	3	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0
	8	0	0	0	0	0	0	0
	9	0	94.1	0	0	0	0	0
	10	0	0	0	0	0	0	0
	11	0	138	0	101	0	64.3	0
	12	0	0	0	0	0	0	0

4.1.3 Case 3, with assumed torque, $M_3 = 10 \cdot 10^6\, N.mm$

Results for the cross-section at the centre of the beams obtained from MathCad 15 software, at each centre position of the concrete and reinforced elements are shown in Tables 12, 13, 14, and 15. Errors of the curvature radius in the directions of the coordinate system O_{xy} and the strain of the grain at the intersection of the axes are shown in Table 11, in which: r_x, r_y are the radius of curvature of the beam, respectively in the x and y directions, and ε_0 is the relative strain of the grain at the intersection of the selected axes (point O). Analysis results are considered convergence when the deviation of the radius, strain of the grain located at the intersection of the selected axes is less than 5%. The results of stress and deformation of the section are accepted at the 4th calculation.

Table 11. The deviation between calculations steps for case 3

Times	Results			Declination (%)		
	r_x	r_y	ε_0	r_x	r_y	ε_0
1	93200	-135000	-0.0009828	-	-	-
2	79040	-105700	-0.0009496	15.19	21.70	3.38
3	78180	-107700	-0.0009609	1.09	-1.89	-1.19
4	78240	-107300	-0.0009585	-0.08	0.37	0.25

Table 12. Strain at the centre of concrete elements at the cross-section between beams

		Column						
		1	2	3	4	5	6	7
Row	1	-9.30E-04	-1.04E-03	-1.23E-03	-1.41E-03	-1.60E-03	-1.79E-03	-1.91E-03
	2	-7.76E-04	-8.88E-04	-1.08E-03	-1.26E-03	-1.45E-03	-1.63E-03	-1.76E-03
	3	-6.61E-04	-7.73E-04	-9.60E-04	-1.15E-03	-1.33E-03	-1.52E-03	-1.64E-03
	4	-4.31E-04	-5.43E-04	-7.29E-04	-9.16E-04	-1.10E-03	-1.29E-03	-1.41E-03
	5	-1.12E-04	-2.23E-04	-4.10E-04	-5.96E-04	-7.83E-04	-9.69E-04	-1.09E-03
	6	2.08E-04	9.61E-05	-9.03E-05	-2.77E-04	-4.63E-04	-6.50E-04	-7.71E-04
	7	5.28E-04	4.16E-04	2.29E-04	4.28E-05	-1.44E-04	-3.30E-04	-4.51E-04
	8	8.47E-04	7.35E-04	5.49E-04	3.62E-04	1.76E-04	-1.06E-05	-1.32E-04
	9	1.08E-03	9.65E-04	7.79E-04	5.92E-04	4.06E-04	2.20E-04	9.83E-05
	10	1.29E-03	1.18E-03	9.96E-04	8.10E-04	6.23E-04	4.37E-04	3.16E-04
	11	1.53E-03	1.41E-03	1.23E-03	1.04E-03	8.53E-04	6.67E-04	5.46E-04
	12	1.69E-03	1.58E-03	1.39E-03	1.21E-03	1.02E-03	8.33E-04	7.12E-04

Table 13. Strain at the centre of reinforcing elements at the cross-section between beams

		Column						
		1	2	3	4	5	6	7
Row	1	0	0	0	0	0	0	0
	2	0	-0.00089	0	0	0	-0.00012	0
	3	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0
	8	0	0	0	0	0	0	0
	9	0	0.000965	0	0	0	0	0
	10	0	0	0	0	0	0	0
	11	0	0.00141	0	0.00104	0	0.000667	0
	12	0	0	0	0	0	0	0

Table 14. Stress value at the centre of concrete elements at the cross-section between of beams, Mpa

		Column						
		1	2	3	4	5	6	7
Row	1	-4.25	-4.76	-5.61	-6.47	-7.32	-8.00	-8.00
	2	-3.55	-4.06	-4.91	-5.76	-6.62	-7.47	-8.00
	3	-3.02	-3.53	-4.39	-5.24	-6.09	-6.94	-7.50
	4	-1.97	-2.48	-3.33	-4.19	-5.04	-5.89	-6.45
	5	-0.51	-1.02	-1.87	-2.73	-3.58	-4.43	-4.98
	6	0.00	0.73	-0.41	-1.27	-2.12	-2.97	-3.52
	7	0.00	0.00	0.00	0.20	-0.66	-1.51	-2.06
	8	0.00	0.00	0.00	0.00	0.00	-0.05	-0.60
	9	0.00	0.00	0.00	0.00	0.00	0.00	0.73
	10	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	11	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	12	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 15. Stress value at the centre of reinforcing elements at the cross-section between of beams, Mpa

		Column						
		1	2	3	4	5	6	7
Row	1	0	0	0	0	0	0	0
	2	0	-174	0	0	0	-16.1	0
	3	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0
	8	0	0	0	0	0	0	0
	9	0	208	0	0	0	0	0
	10	0	0	0	0	0	0	0
	11	0	250	0	219	0	138	0
	12	0	0	0	0	0	0	0

4.2 Analysis Results of ANSYS

Analysis results for RC beams with ANSYS Workbench 2019 R3 are shown below.

Fig. 11, 12, and 13 show the values of stress in the x-direction and images of cracks along the RC beams respectively for cases 1,2, and 3. The moment value in the first, second and third cases has been replaced by the corresponding concentration force when simulating with ANSYS, and the distance from the bearing to the concentrated force is 1000mm, the corresponding forces are P1=1000 N, P2 = 5000 N, P3 = 10 000 N. The concentrated forces are arranged at an angle of 10 degrees from the vertical axis and downwards.

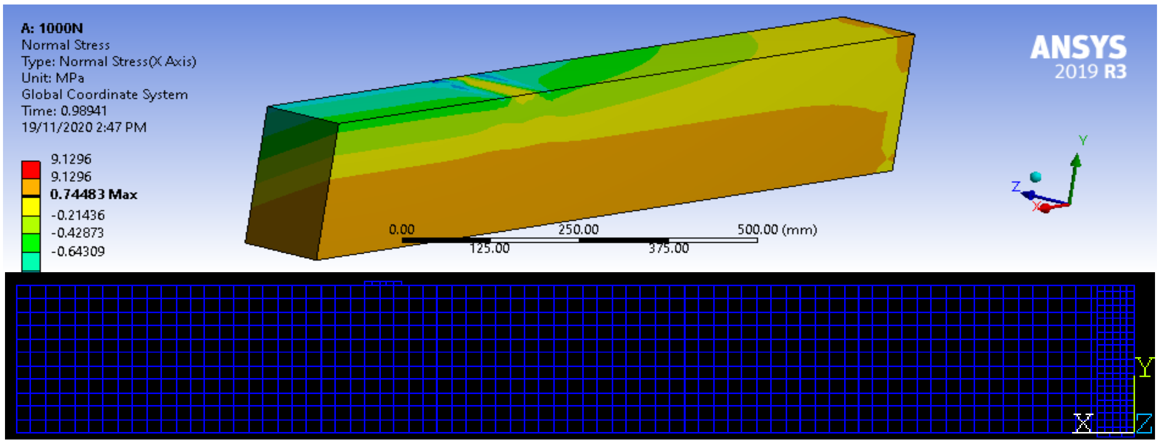


Fig. 10 Stress spectrum of case 1 (M=106 N) and cracks along beam length

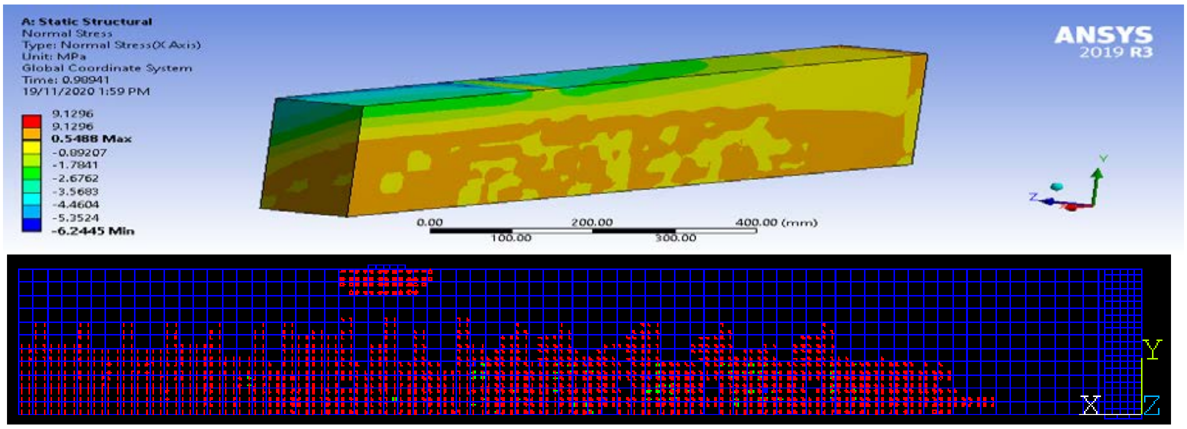


Fig. 11 Stress spectrum of case 2 (M=5.106 N) and cracks along beam length

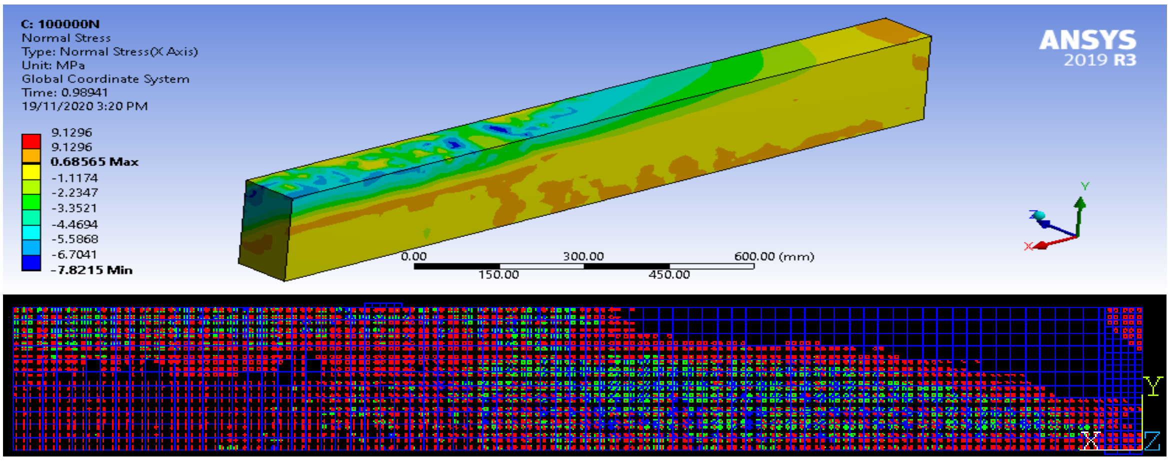
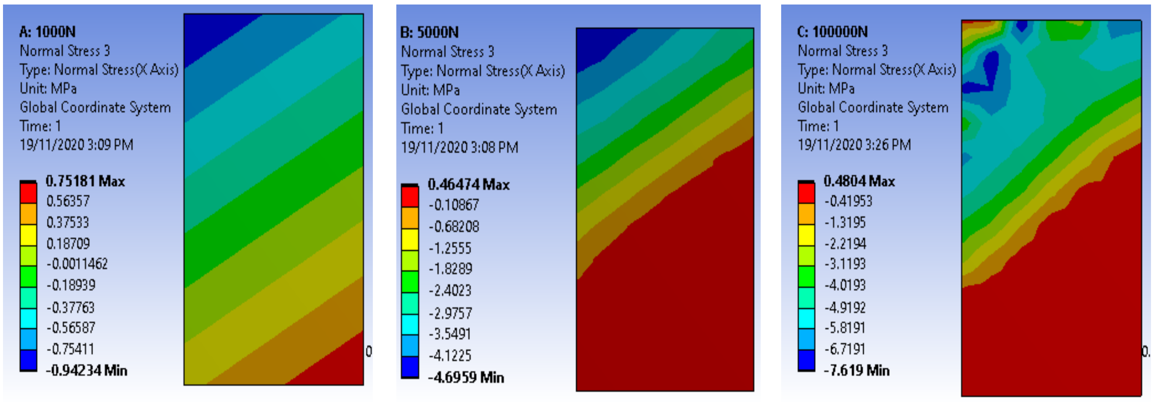


Fig. 12 Stress spectrum of case 3 (M = 5.106 N) and cracks along beam length



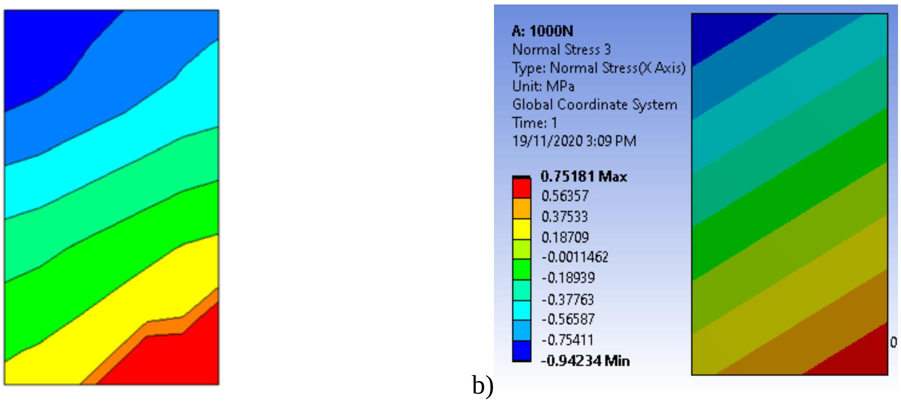
The case 1 The case 2 The case 3

Fig. 13. Distribution of Stress on the cross-section between beams

5. DISCUSSION

The comparison of the two methods is set out below.

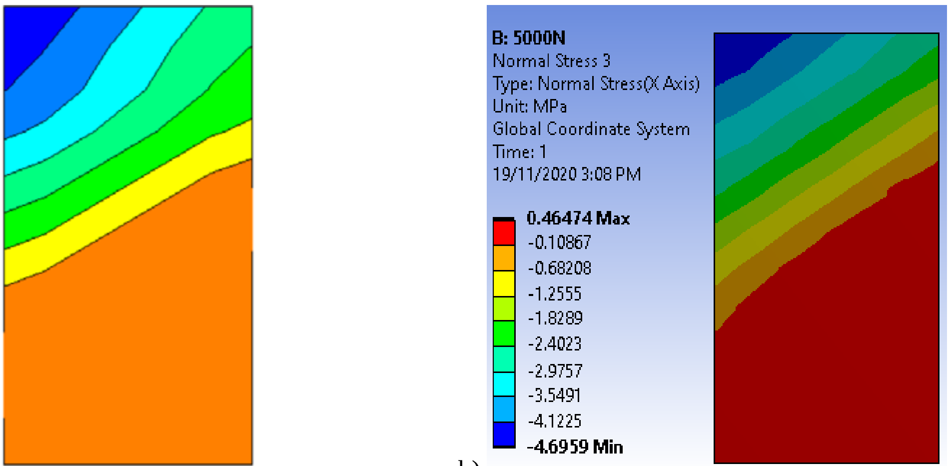
Case 1:



a) b)

Fig. 14. Stress spectrum on the cross-section for case 1
a) Recommended method. b) ANSYS method

Case 2:



a) b)

Fig. 15. Stress spectrum on the cross-section for case 2
a) Recommended method. b) ANSYS method

Case 3:

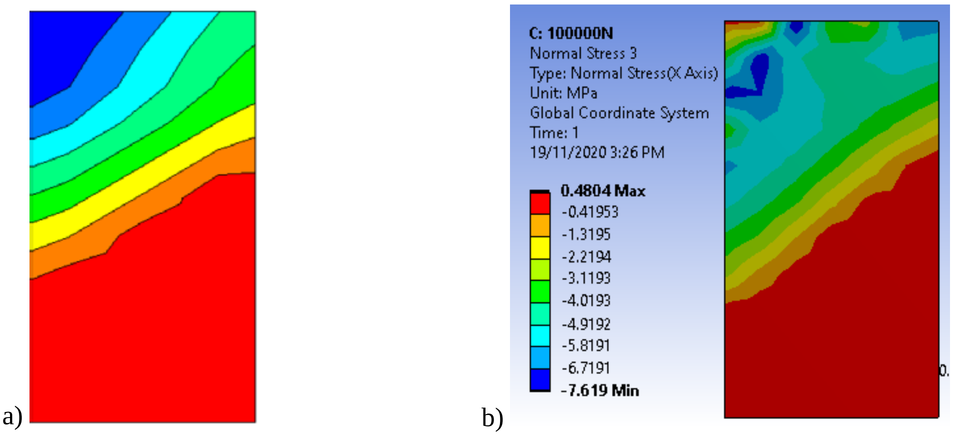


Fig. 16. Stress spectrum on the cross-section for case 3
a) Recommended method. b) ANSYS method

Observing the stress spectra of the above three cases indicates that there are similarities between the two analytical methods. The stress values of the two methods are similar; the stress values are different but not significantly so. The stress spectrum of the methods show that:

- The shape of the neutral axis on the cross-section of the two methods is similar.
- The positions of the neutral axis on the cross-section are the same
- The shape and area of the pull or compression range are the same
- The positions of the compression and tensile range in the cross-section with maximum stress are the same.
- The position and length of the crack in the cross-section are the same.

Tables 4 (case 1), Table 9 (case 2), and Table 14 (case 3) show the stress results of the concrete elements corresponding to three different load cases. Table 4 (case 1) shows non-zero stress values, which indicates that the crack did not appear. Table 9 (case 2) and Table 14 (case 3) show zero stress values, indicating that a crack appeared. In these two cases, the proposed method also shows the length of the crack. In case 1, because the value of the load is small, the stress-strain relationship is linear, so the crack does not appear (Fig. 11). In case 1, it is understood that the compressive stress of the concrete is in the compression range and that the reinforcement (tensile and compression) has not yet reached its ultimate value. In case 2, when the load value has been increased, the stress-strain relationship is nonlinear and cracks appear (Fig. 12). In case 3, the maximum load corresponding to the ultimate crack occurs, and the RC beam begins to fail (Fig. 13). On the other hand, the cracks on the left side of the cross-section are shorter than those on the right side because they are further from the load direction. Thus, the results are close to reality, meaning that the method achieves reasonable accuracy.

The results of the finite element method - ANSYS - also gave results similar to the proposed method (Fig. 14,15,16).

Table 16. Table of summary and comparison of maximum compressive stress of concrete on the cross-section between beams.

	Stress (MPa)		
Case	Analysis method	ANSYS	difference
1	-0.77	-0.94	0.17
2	-4.37	-4.69	0.32
3	-8.00	-7.61	0.39

Table 16 summarises the results of the three maximum compressive stresses of the three different load values and compares the results between the two methods; the difference is not significant. The deviations between the two methods for case 1, case 2, case 3 are 0.17, 0.32, and 0.39, respectively. This confirms that the accuracy and reliability of the proposed method are acceptable.

6. CONCLUSION

The study proposed an analytical method for determining the stress-strain state of an RC concrete beam cross-section subjected to biaxial bending. The results were compared and verified with the finite element method. From the results, the following conclusions are drawn:

1. The method of stress-strain state analysis of the cross-section of RC beams proposed in this study is easy to implement. Furthermore, accurate and

reliable results were obtained by integrating this method with MathCad15 and OriginPro.
2. The method proposed in this study determines crack shape, location, and length of the crack that are proven to be correct.
3. The shape, position of the neutral axis; the shape, area of the tensile, and compressive parts from the proposed method results are in good agreement with reality.
4. The results of the final analysis allow evaluation of the accuracy of the material selection and the initial layout of the rebars.
5. The proposed analytical method can also be used for different cases, such as various values of load and cross-sectional shapes.

7. REFERENCES

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