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RESEARCH

Reliability assessment of circular steel arches with elastic restraints using hybrid ANN-MCS technique

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Abstract
The circular steel arches are large-span structures, which are shaped into a circular or semicircular form. The circular steel arch is widely used in bridges, tunnels, architectural design, industrial and warehouse buildings, and aqueducts. Circular steel arches are known for their strength and durability, making them a popular choice in architecture and civil engineering. The safety of circular steel arches bearing radial load with elastic rotational restraints depends on material properties, geometric dimensions, and boundary conditions. The objective of this research is to perform a reliability assessment of the in-plane elastic buckling critical load of circular steel arches with elastic rotational restraints considering random input parameters. For that, the Artificial Neural Network (ANN) algorithm is used to construct a model for estimating the in-plane elastic buckling critical load of the circular steel arches, while Monte Carlo Simulation (MCS) is used to simulate the in-plane elastic buckling critical load and assess structural reliability. The calculated results of the proposed model are compared with FORM, SORM, and MCS. Eventually, the influence of random input parameters on the reliability of circular steel arches is evaluated using the first order and total Solol's indices.

Keywords Reliability assessment · In-plane elastic buckling · Critical load · ANN-MCS · Circular steel arches

Introduction

The circular steel arche is widely used in bridges, tunnels, architectural design, industrial and warehouse buildings, and aqueducts. Circular steel arches are known for their strength and durability, making them a popular choice in architecture and civil engineering. The safety of circular steel arches bearing radial load with elastic rotational restraints depends on material properties, geometric dimensions, and boundary conditions (Nguyen, 2020). In reality, these factors are random, so assessing the safety probability of circular steel arches bearing radial load with elastic rotational restraints has scientific and practical significance.

Analytical studies of the in-plane buckling is a topic of research interest to many scientists (Gjelsvik & Bodner, 1962; Pi et al., 2002; Timoshenko & Gere, 2009). Investigates the non-linear in-plane buckling of pin-ended shallow circular arches with elastic end rotational restraints under a central concentrated load (Pi et al., 2008). investigation of non-linear buckling and postbuckling analyses of pin-ended shallow circular arches subjected to a uniform radial load and which have equal elastic rotational end-restraints (Pi & Bradford, 2009). Analytical study of the non-linear elastic in-plane buckling and postbuckling behaviour of pin-ended shallow circular arches having unequal elastic rotational end restraints under a central concentrated radial load (Pi & Bradford, 2012).

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Reliability probability is the safety index of the structures with random input parameters. In recent years, there have been safety probability assessment methods (reliability) of a structure such as the First-order probability method—FORM, the Second-order probability method—SORM, the Subset simulation method, Time-dependent reliability analysis, and Monte Carlo Simulation—MCS. Can find studies on the reliability probability for steel structures in general and steel arch structures, in particular, using these methods (Ha, 2019; Kaveh & Zaerreza, 2022; Ngoc-Long & Ha, 2020; Tran & Nguyen, 2020). However, these methods often have limitations and use a lot of resources and computing time.

The combination of ANN model and MCS method will bring many benefits and potential in structural analysis and calculation. The ANN-MCS hybrid model can leverage the strengths of both techniques to improve the accuracy, reliability, and efficiency of modeling and analysis. ANNs are capable of learning complex relationships and patterns from data, while MCSs enable probabilistic modeling and quantification of uncertainty. By combining these techniques, we can improve the accuracy of prediction and simulation by incorporating both deterministic and probabilistic elements (Cardoso et al., 2008; Papadrakakis & Lagaros, 2002; Papadrakakis et al., 1996).

This paper presents a reliability assessment of the in-plane elastic buckling critical load of circular steel arches with elastic rotational restraints. For that, the ANN algorithm is used to build a model to estimate the in-plane elastic buckling critical load with elastic rotational restraints, while MCS is used to simulate critical load values and reliability probability evaluation. The calculation results of the proposed model are compared with other traditional reliability methods such as MCS, FORM, and SORM. Finally, the effects of random input parameters on the reliability of in-plane circular steel arches is evaluated using the first order and total Solol's indices.

Theoretical framework

The in-plane elastic buckling critical load of circular steel arches bearing radial with elastic rotational restraints

Considering circular arc AB bearing radial load is shown in Fig. 1. Called $u_0, w_0, u_1, w_1,$ are directional displacement x and y of A and B, respectively; u and w are tangent displacement and centripetal displacement of B. We have the following relationship.

$$x = r\sin\varphi - u_1 + u_0 \tag{1}$$

$$y = \mathrm{r} - r\cos\varphi - v_1 + w_0 \tag{2}$$

$$w = u_1\sin\varphi + v_1\cos\varphi \tag{3}$$

Bending moment at B_1 in deformed state can be expressed by

$$M = M_0 + T_0x + N_0y + qry - \frac{q}{2}\left(x^2 + y^2\right) \tag{4}$$

Based on Eqs. (1–4), the expression of bending moment can be re-written as follows.

$$M = A + B\sin\varphi + C\cos\varphi + qrw \tag{5}$$

where $A = M_0 + N_0r; B = T_0 + qru_0; C = -\left(N_0r + qrw_0\right)$.

According to Leu (2005), the differential equation of the centripetal displacement can be expressed by

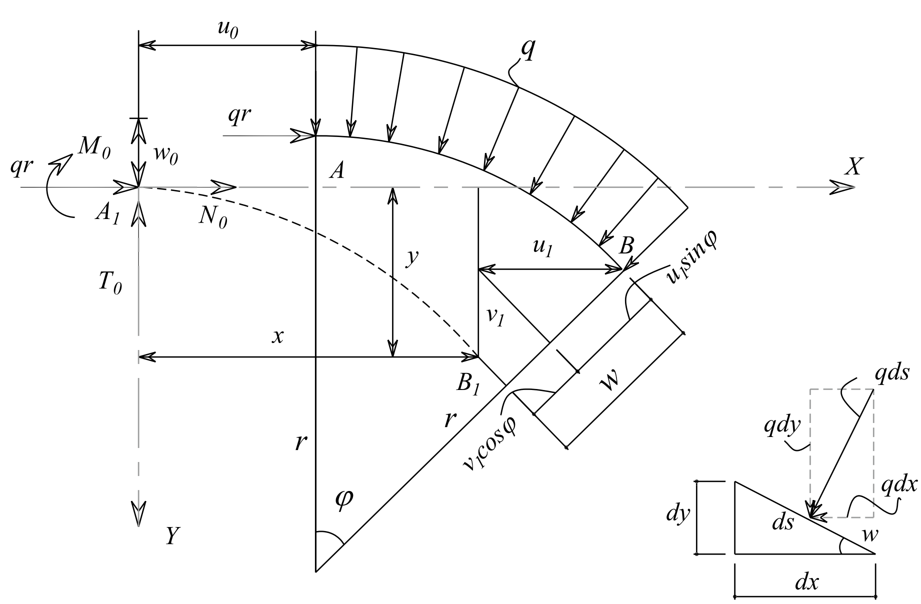

Fig. 1 Circular arc AB bearing radial load

$$\frac{d^2w}{ds^2}+\frac{w}{r^2}=-\frac{M}{EI}\text{or}\frac{d^2w}{d\varphi^2}+w=-\frac{Mr^2}{EI} \quad (6)$$

Substituting Eq. (5) into Eq. (6), obtained:

$$\frac{d^2w}{d\varphi^2}+w=-\frac{r^2}{EI}(A+B\sin\varphi+C\cos\varphi+qrw) \quad (7)$$

or

$$\frac{d^2w}{d\varphi^2}+k^2w=-\frac{r^2}{EI}(A+B\sin\varphi+C\cos\varphi) \quad (8)$$

where $k^2=\frac{qr^2}{EI}+1$

By integral of Eq. (7), we obtain the centripetal displacement w at angular coordinate φ as

$$w=D_1\sin k\varphi+D_2\cos k\varphi+\frac{Ar^2}{k^2EI}+\frac{Br^2\sin\varphi}{(1-k^2)EI}+\frac{Cr^2\cos\varphi}{(1-k^2)EI} \quad (9)$$

Applying Eq. (9) for circular in Fig. 1, combined the boundary conditions:

$\varphi=0\rightarrow w=0$; (at arch crown) $D_2=0$;

$\varphi=\alpha\rightarrow w=0$ (at arch bottom),

Equation (9) is re-written as:

$$D_1\sin k\alpha+\frac{Br^2\sin\alpha}{(1-k^2)EI}=0; \quad (10)$$

If C is the stiffness of elastic constraining, the bending moment at the arch bottom is equal to $B\sin\alpha$, the arch bottom angular is determined by $C.B\sin\alpha$. We obtained:

$$kD_1\cos k\alpha+\frac{Br^2\cos\alpha}{(1-k^2)EI}=CB\sin\alpha; \quad (11)$$

Combining Eq. (9) and Eq. (10), the following equation is obtained:

$$\sin k\alpha\left[\cot\alpha-k\cot k\alpha-\frac{cEI}{r^2}(1-k^2)\right]=0 \quad (12)$$

Equation (7) was used to determine the coefficient k, this is a transcendent equation. Therefore, in this study, we choose the ANN-MCS method to predict critical load as:

$$q_{cr}=(k^2-1)\frac{EI}{r^3} \quad (13)$$

Safety conditions

The safety conditions of the in-plane elastic buckling critical load of circular steel arches subjected to radial load with elastic rotational restraints is determined according to Eq. (8). The safety condition is rewritten as follows:

$$q\leq q_{cr} \quad (14)$$

Deterministic model

The deterministic model represents the relationship between the input and output parameters of the safety condition (9), expressed by the following form.

$$q\leq q_{cr}(\alpha,E,C,D,d,r) \quad (15)$$

The stochastic model

The stochastic model in this study was built based on a deterministic model with random input parameters (ω). It is written as

$$q\leq q_{cr}(\alpha(\omega),E(\omega),C(\omega),D(\omega),d(\omega),r) \quad (16)$$

Artificial Neural Network (ANN)

Artificial Neural Network (ANN) is machine learning and artificial intelligence technical, that is widely used in engineering and technology (Hosseini et al., 2023; Kaveh & Khavaninzadeh, 2023; Kaveh et al., 2023; Nguyen et al., 2021, 2023a). The structures of an Artificial Neural Network backpropagation and Levenberg–Marquardt algorithm have three layers (input layer, hidden layer, and output layer) and shown in Fig. 2. Its mathematical representation has the form.

$$X\in R^D\rightarrow Y\in R^1 \quad (17)$$

$$f(X)=f_0\big(b_2+W_2\big(f_h\big(b_1+W_1X\big)\big)\big) \quad (18)$$

where b_1, W_1 and f_h bias vectors, weight matrix, and active function of hidden layers, respectively; b_2, W_2 and f_0 bias vectors, weight matrix, and active function of output layers, respectively.

In this study, the hidden layer activation function was used the nonlinear function *tansig*, while the linear function *purelin* was used for the output layer, as demonstrated in Fig. 3.

Monte Carlo simulation (MCS)

The probability of failure structures in vector of random variables $X=\left[\boldsymbol{X}^R,\boldsymbol{X}^S\right]$ is defined by the following relationship:

$$\overline{p_f}=\text{Prob}\{G(\boldsymbol{X})\equiv R-S\leq 0\}=\int_{G(X)\leq 0}f(\boldsymbol{X})d\boldsymbol{X} \quad (19)$$

Then probability of reliability structures is defined by.

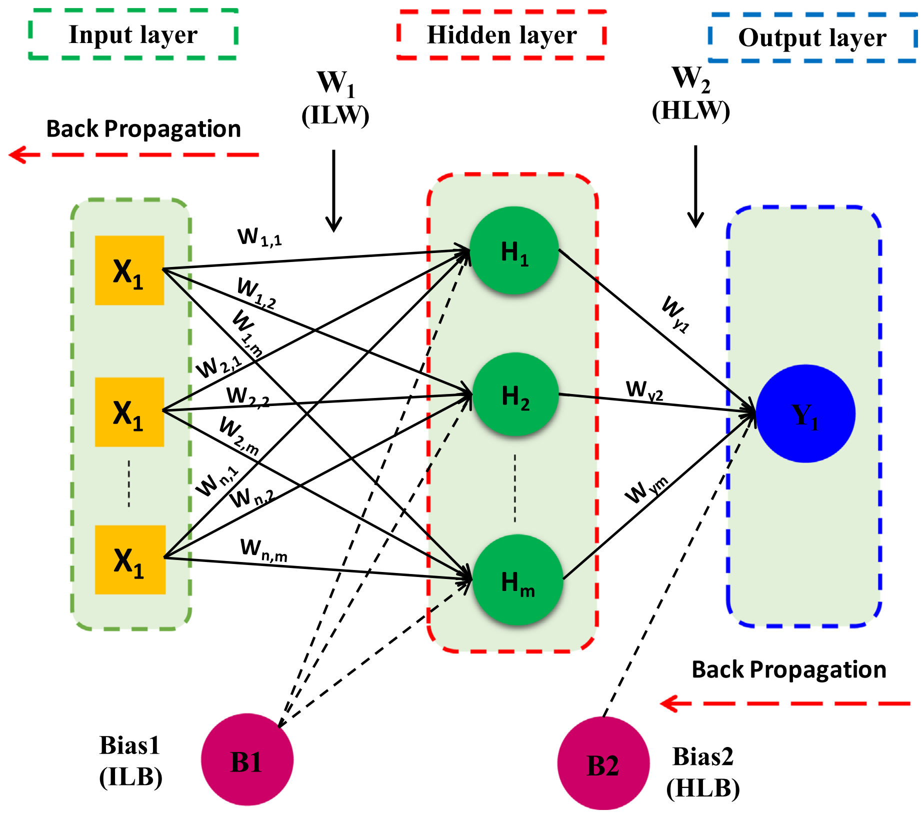

Fig. 2 Neural network—back propagation algorithm

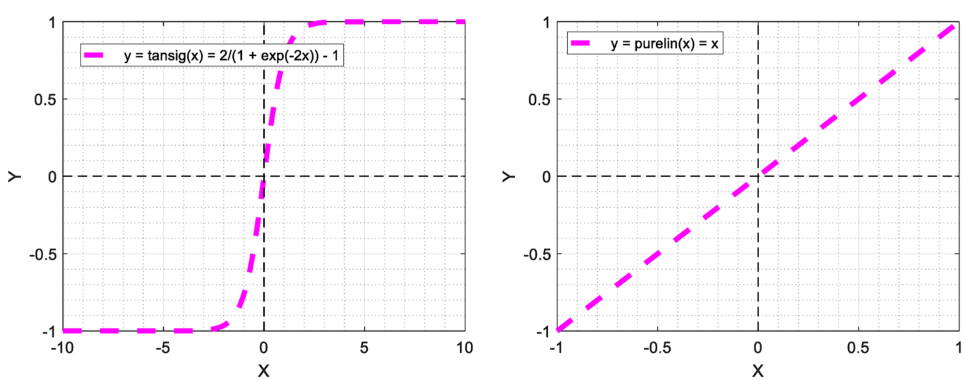

Fig. 3 Active function *tansig* and *purelin*

$$p_s = 1 - \overline{p_f} \quad (20)$$

where $\overline{p_f}$, R, and S are probability of failure, resistance of structure, and actions (loads), respectively.

According to the law of large numbers the classical Monte Carlo estimator of the probability of failure in Eq. (14) has the following form.

$$\overline{P}_f = \frac{1}{N}\sum_{i=1}^{N} I(X_i)I(X_i) = \begin{cases} 1\, for\, G(X_i) \leq 0 \\ 0\, for\, G(X_i) > 0 \end{cases} \quad (21)$$

where $f(X)$ is probability distribution function; g (X) is sampling function.

Reliability assessment using hybrid Artificial Neural Network (ANN) and Monte Carlo simulation (MCS)

Reliability assessment using hybrid ANN-MCS in this study is performed according to the following steps (Papadrakakis & Lagaros, 2002).

Step 1. Preparing data includes input random variables and response functions.
Step 2. Generate input data samples and calculate critical loads.

Step 3. Training, testing, and validation of generate input data samples in step 2 using the ANN model.
Step 4. Reliability assessment using MCS.
Step 5. Calculate Sobol' indices using MCS.

The flowchart of reliability assessment structure steps using hybrid ANN-MCS is shown in Fig. 4.

Reliability assessment of circular steel arches bearing radial load with elastic rotational restraints

Input parameters

Considering the circular steel arches bearing radial load with elastic rotational restraints are shown in Fig. 5. The safety condition of circular steel arches according to Eq. (9). The mean and statistical properties of random input parameters are shown in Table 1. These random input parameters generate 10.000 input samples for the ANN model.

Reliability assessment

To find the ANN model structure is best predictive performance critical load of circular steel arches bearing radial load with elastic rotational restraints. A series of tests have been investigated with an 80% training data ratio and 20% testing data ratio, and the number of hidden layers is 4, 6, 8, 10, and 14, respectively. Prediction performance is evaluated based on the mean squared error (MSE). The results of the tests are shown in Table 2.

Table 2 shows that the best predictive performance of the ANN model with MSE = 0.000146 corresponding to a 10-layer hidden structure and a training time of 245 s, using an Intel® Core™ i7-7500U Processor CPU system (Fig. 6).

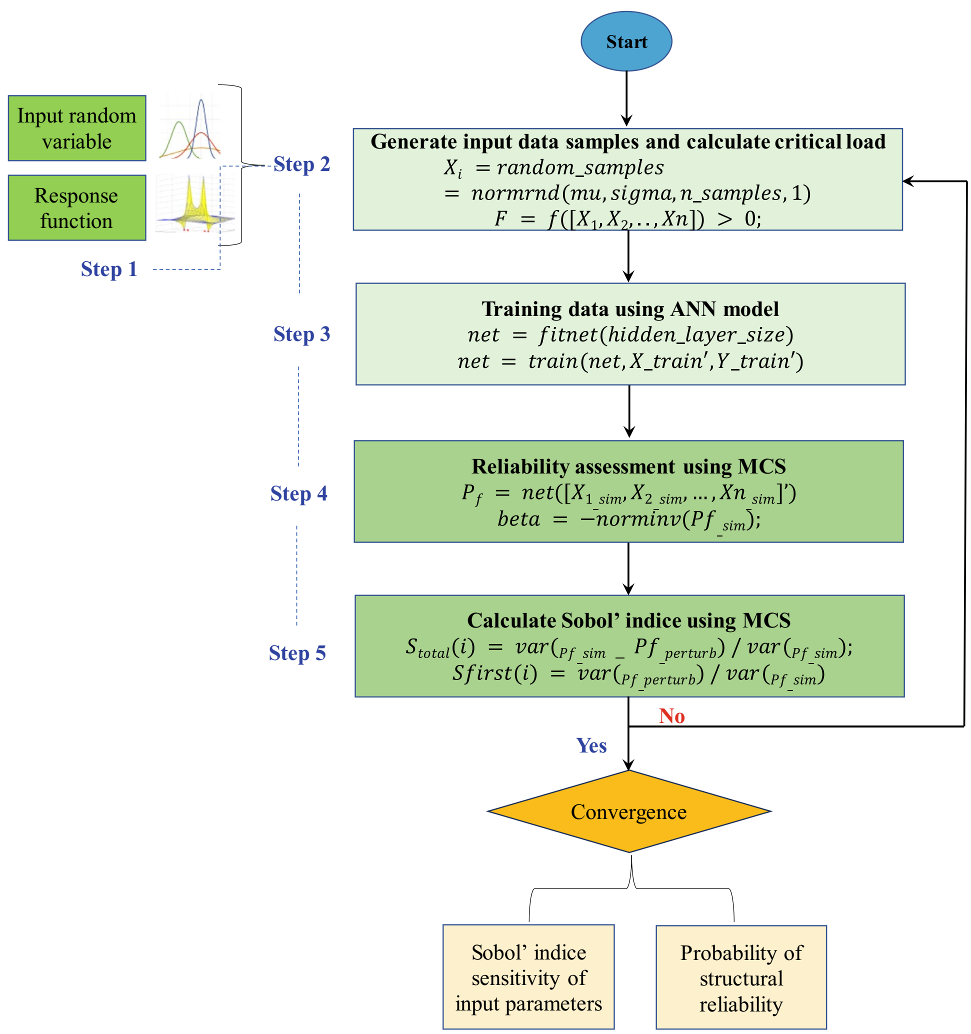

Fig. 4 The reliability assessment structure using hybrid ANN-MCS

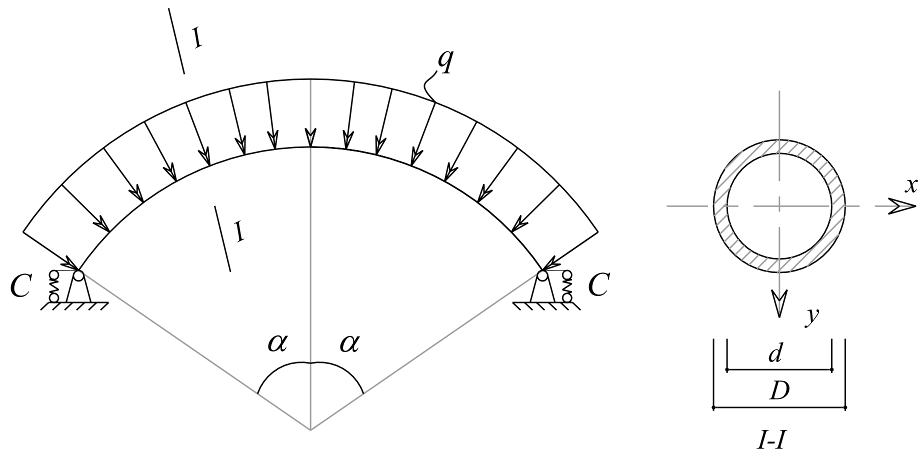

Fig. 5 The circular steel arches bearing radial load with elastic rotational restraints (left), the cross-section of the arch (right)

Table 1 Random input parameters and statistical properties

Variable	α(degree) (X_1)	E(GPa) (X_2)	C(kN.m) (X_3)	D(mm) (X_4)	d(mm) (X_5)	r(mm) (X_6)
Nominal	35.0	210	10,320.78	500	400	5000
Mean/nominal	1.0	1.1	1.0	1.0	1.0	1.0
COV	0.05	0.06	0.05	0.05	0.05	0.05
Distribution	N*	LN*	N	N	N	N
Refs.	Ellingwood et al., (1982)	Bartlett et al., (2003)	Ellingwood et al., (1982)	Ellingwood et al., (1982)	Ellingwood et al., (1982)	Ellingwood et al., (1982)

*N: Normal, LN: Lognormal

Table 2 Prediction performance of the ANN model

Hidden layer neurons	MSE	Times (s)
4	0.0020164	52
6	0.0013509	102
8	0.0020897	204
10	0.0001461	245
14	0.0001641	624

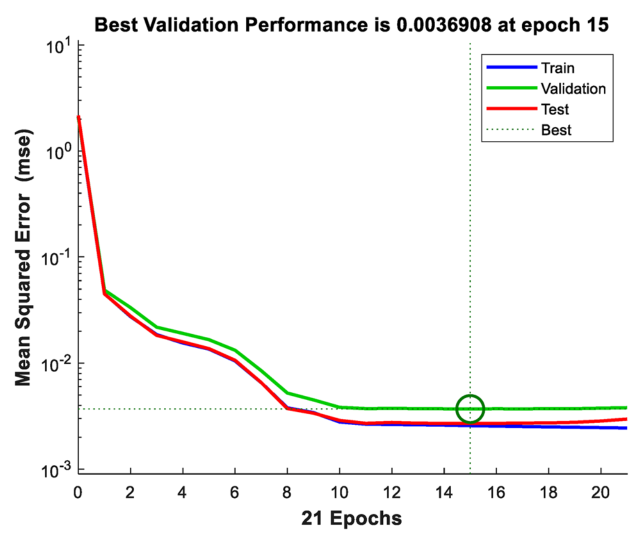

Fig. 6 Best validation performance of ANN

To assess the reliability of circular steel arches with elastic rotational restraints, the ANN-MCS algorithm is implemented with 10.000 simulations converging at a rate of 1.5%. The resulting outcomes include the probability of failure (P_f) and the reliability index (β), as presented in Table 3. The distribution chart of the reliability index β as shown in Fig. 7. These findings have been compared against traditional MCS, FORM, and SORM techniques. The comparative results demonstrate that the proposed ANN-MCS algorithm for evaluating the reliability probability of varying cross-sectional steel columns is indeed reliable.

Table 3 Probability safety and reliability index of circular steel arch

	ANN-MCS	MCS	FORM	SORM
Probability of failure (P_f)	0.06369	0.038895	0.03888	0.03889
Reliability index (β)	1.5245	1.7637	1.7638	1.7638

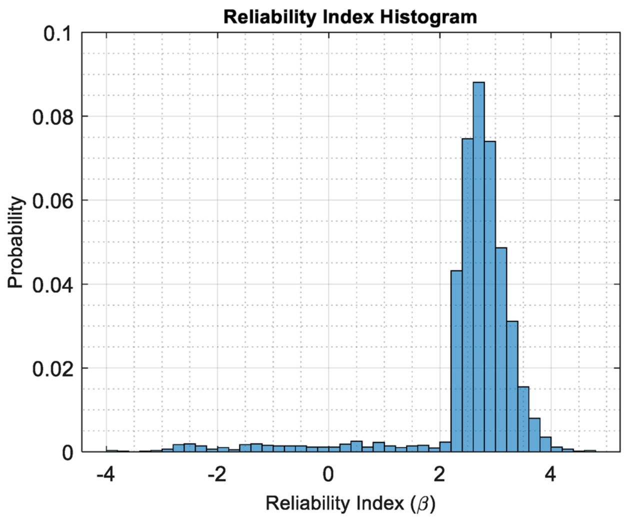

Fig. 7 Histogram of reliability index after 10.000 simulations

Table 4 Effect of random input parameters after 10.000 simulations

Input variables	First order Sobol' indices	Total order Sobol' indices
$\alpha(X_1)$	1.0010	0.0001
$E(X_2)$	1.0008	0.0000
$C(X_3)$	1.0299	0.0251
$D(X_4)$	1.1741	0.7059
$d(X_5)$	0.9479	0.0798
$r(X_6)$	0.9157	0.1891

Effects of random input parameters on reliability analysis

The randomness of input parameters (Table 1) is investigated. After 10.000 simulation, the First-order Sobol' indices and Total-order Sobol' indices are shown in Table 4 and Fig. 8. It can be observed that the First-order Sobol' indices of random input parameters have equal contributions with between $15\,and\,17\%$. Whereas the Total-order Sobol' indices have seen a significant variation between random input parameters. Specifically, the outer diameter of cross section $D(X_4)$ is the most sensitive parameter (62%), followed by radius of circular steel arches $r(X_6)$ (24%) and inner diameter of cross section $d(X_5)$ (10%). Meanwhile, the other parameters are less sensitive with smaller than 5%.

Effects of elastic restrain coefficients on reliability of steel arch

In this section, a wide range of elastic restrain coefficients of the arch is considered. The stiffness of restrain varies from zero (i.e., pinned connection) to infinity (i.e., fixed end), as shown in Table 5. Results of reliability histograms of the steel arch for various restrain conditions are shown in Fig. 9. It is found that the probability of safety is gradually increasing with an increment of restrain stiffness. The reliability index (β) is increased from -1.4881 for pinned end to 2.1622 for fixed end. This emphasizes again that the boundary condition affects structural safety significantly when random variables are considered.

Fig. 8 First and Total order Sobol' indices of random input parameters

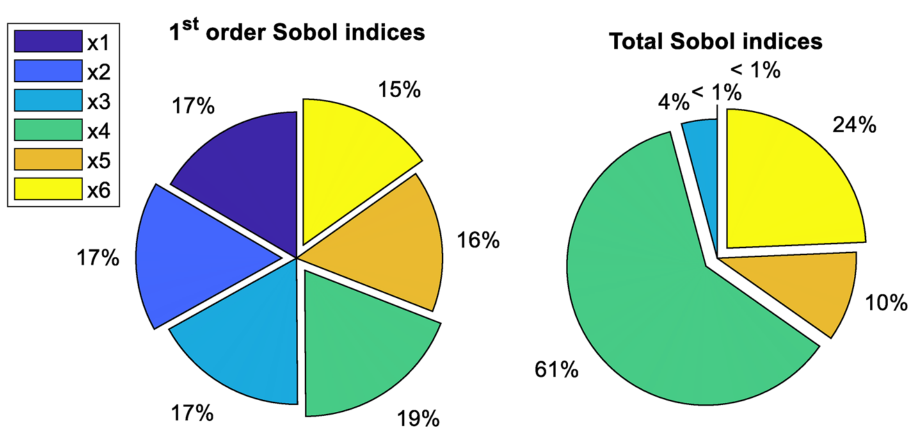

Table 5 Variation of elastic restrain stiffness

C(kN.m)	Case 1	Case 2	Case 3	Case 4
Nominal	0	1.03e4	2.06e8	*inf*
Mean/nominal	1.0	1.0	1.0	1.0
COV	0.05	0.05	0.05	0.05
Distribution	N	N	N	N
Probability of failure (P_f)	0.9316	0.0648	0.0622	0.0153
Reliability index (β)	− 1.4881	1.5155	1.5366	2.1622

Conclusions

This paper proposed hybrid ANN-MCS algorithm for reliability assessment of circular steel arches subjected to radial loads considering elastic rotational restraints. For that, the stochastic model was determined using the randomness of input parameters. The ANN algorithm was developed to construct a model for estimating the critical load of the circular steel arches, while MCS was employed to generate various scenarios of critical loads and assess the structural reliability. The calculated results were verified with traditional FORM, SORM, and MSC methods. In addition, the effects of random input parameters on the reliability probability of circular steel arches were also evaluated using Sobol' sensitivity index. The conclusions are drawn as follows:

(1) A stochastic algorithm using hybrid ANN-MCS is proposed for in-plane elastic buckling load and reliability analysis of circular steel arches with elastic rotational restraints. Random variables including structural and material properties as well as restraining stiffness are considered in the process.
(2) The outer diameter of cross section $D(X_4)$ is the most sensitive parameter (62%), followed by radius of circular steel arches $r(X_6)$ (24%) and inner diameter of cross section $d(X_5)$ (10%).
(3) The probability of safety is increasing with an increment of restrain stiffness. The reliability index (β) is increased from the case of pinned-end to fixed-end condition. The boundary condition affects structural safety significantly.

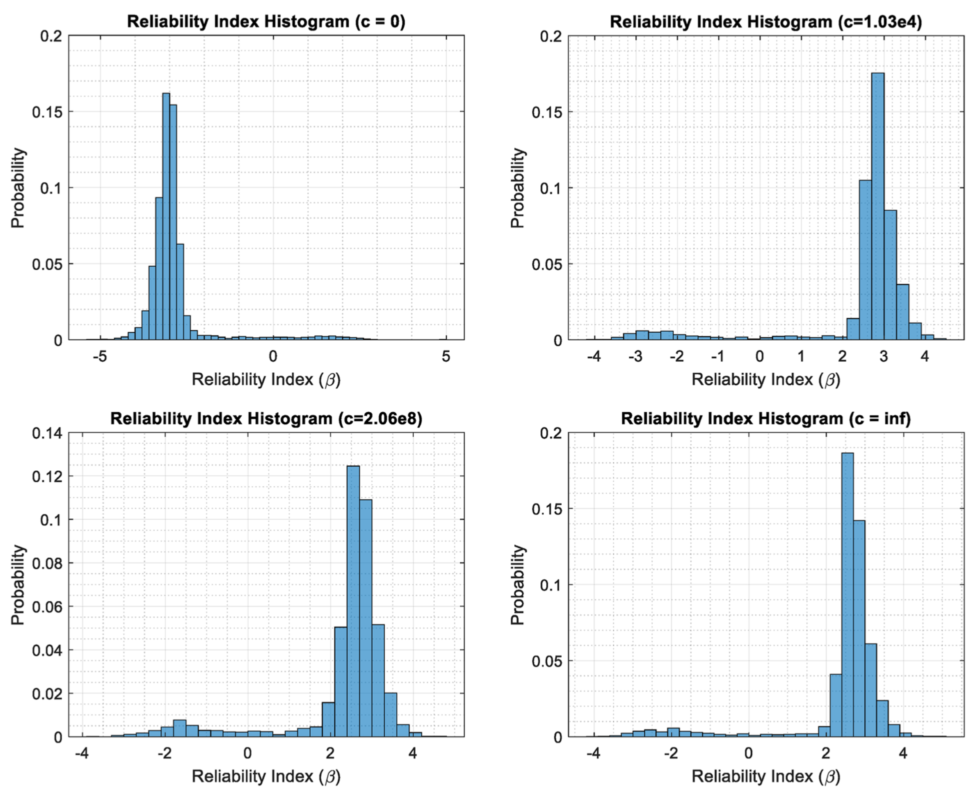

Fig. 9 Reliability index histogram for various elastic restrain stiffness

Author contribution S-MN: Methodology, Writing-Review and Editing, Formal analysis, N-LT: Writing-Review and Editing, Formal analysis, X-TP: Writing-Review and Editing. X-HN: Writing-Review an Editing. D-DN: Formal analysis, Writing–Original Draft, Writing–Review and Editing, Supervision. T-HN: Conceptualization, Software, Writing-Original Draft, Writing-Review and Editing.

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Data availability The data used to support the findings of this study are included in the article.

Declarations

Conflict of interests The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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