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# Indirect Reinforcement of Reinforced Concrete Elements as a Means of Protecting a Constructive System from a Progressive Collapse

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**Abstract**: A variant of the diagram of static-dynamic deformation of compressed concrete, reinforced by indirect reinforcement of statically indefinable structural systems in bending reinforced concrete elements, is constructed. The characteristic parametric points of the static section of the diagram to the level of the operational load are determined by a three-line diagram using the proposal of A G Tamrazyan. The linear dynamic part of the deformation caused by a sudden structural reorganization of the constructive system is described by the deformation model of G A Geniev. The increments of dynamic forces in an arbitrary cross section of a reinforced concrete element with indirect reinforcement n times of a statically indefinable structural system are determined by a quasistatic method on an energy basis. Numerical analysis of a two-span continuous beam loaded with a given operational load and a special effect in the form of a sudden removal of one of the supports shows the effectiveness of indirect reinforcement to protect the structural system from progressive collapse.

## 1. Introduction

It is known that indirect reinforcement of compressed reinforced concrete elements significantly increases their strength by reducing transverse deformations [1-3]. In this regard, it is logical to assume that the use of indirect reinforcement in bending, eccentrically compressed and compressed elements can have a positive effect on the redistribution of power flows in a structural system during its structural adjustment, which is required when conducting a computational analysis of progressive collapse [4]. Such a restructuring may be caused by the sudden removal of one of the supporting elements. Thus, it is possible to increase the protection of the structural system against progressive collapse under the specified effect. To assess the effectiveness of such protection, this paper presents a quantitative analysis of the change in dynamic strength and dynamic loading time in the cross section of a loaded flexural reinforced concrete element, reinforced by indirect reinforcement during its dynamic loading.

## 2. Method

To solve the formulated problem, we use the energy approach [5] and elements of the theory of plasticity of concrete and reinforced concrete by G A Geniev [6] to determine the parameters of static-dynamic deformation of a reinforced concrete element of a structural system in the limiting and transcendental states. A number of researchers [7-17] were engaged in the construction of concrete deformation diagrams reinforced with indirect reinforcement. As shown in the study [18], quite satisfactory convergence of experimental and theoretical results can be obtained by using the linearized diagram given in [19]. Let us construct a version of this diagram for the case under consideration of static-dynamic deformation of concrete reinforced with nets, highlighting a section of static deformation - during short-term loading of concrete and a section of dynamic loading - with fast flowing (shock) loading.

For the static section of the concrete deformation diagram with indirect reinforcement (0 – a – b), one can accept the bilinear dependence of the form:

If $0 \le \varepsilon_b \le \varepsilon_{b23}$
$$\sigma_b = \varepsilon_b E_b \tag{1}$$

If $\varepsilon_{b23} \le \varepsilon_b \le \varepsilon_{b23}$
$$\sigma_b = \left(0.4\frac{\varepsilon_b - \varepsilon_{b23}}{\varepsilon_{b33} - \varepsilon_{b23}} + 0.6\right) R_{b3} \tag{2}$$

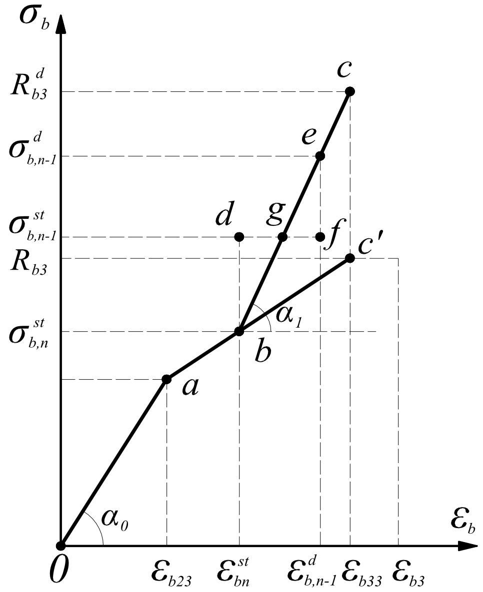


**Figure 1.** Linearized diagram "$\sigma_b - \varepsilon_b$" static-dynamic deformation of concrete.

The construction of the dynamic section of the concrete deformation diagram with indirect reinforcement 2 - 3 was performed using dependencies [6]. Determination of the dynamic strength of compressed concrete, reinforced with indirect reinforcement meshes, $R_{b3}^d$, can be performed by analogy with the problem of determining the dynamic strength of unreinforced concrete [18,19], assuming that, like for the static section of the "$\sigma_b - \varepsilon_b$" diagram, physicomechanical processes in such a reinforced concrete can be represented by the deformation model of the theory of plasticity by G A Geniev. Then the relationship between the intensity of the relative deformation $\gamma$, the relative dynamic tensile strength $\varphi$ and the dynamic action time $\xi$ can be represented by a nonlinear differential equation of the form [5,6]:

$$\frac{\partial \gamma}{\partial \xi} + \gamma\left(1 - \frac{1}{2}\right)\gamma = \frac{1}{2}\varphi \tag{3}$$

Where, in the uniaxial stress state of compressed concrete with indirect reinforcement, the value of the relative parameters γ, φ, and ξ are determined by the expressions:

$$\gamma = \frac{\varepsilon_b}{\varepsilon_{b03}}, \quad \varphi = \frac{\sigma_b}{R_{b3}}, \quad \xi = \omega t \tag{4}$$

here $\omega = \frac{G_0}{\eta_1}$; $G_0$ - is the long-term shear modulus of concrete, $\eta_1$ - is the coefficient of viscosity of concrete, determined experimentally.

Since the static-dynamic diagram considers the section of dynamic loading of loaded concrete, there are no experimental data for determining the parameter ω, there are no experimental data for such a loading mode. In the first approximation for the quantitative analysis of the dynamic strength of reinforced concrete we will use the average value of this parameter for heavy concrete [6]. In the considered problem, the solution of equation (3) will be of interest to us at $\varphi > 1$. Taking $\gamma = 0$ and $\xi = 0$ as initial conditions, we can write:

$$\gamma = \varphi / \left[ (1 + (\varphi - 1)^{1/2}) \times (ctg(\varphi - 1)^{1/2} \ )0.5\xi \right] \tag{5}$$

Solving (3), with $\xi = \xi_d$, using the deformation criterion of the limit state of reinforced concrete $\gamma = 1$, it is easy to establish a connection between the dynamic tensile strength of concrete reinforced with indirect reinforcement and the maximum allowable dynamic effect time, i.e. the application time of the dynamic addition (point b on the "$\sigma_b - \varepsilon_b$"diagram before concrete breakdown (point c)):

$$\xi_d = \frac{2arcctg\sqrt{\varphi_d - 1}}{\sqrt{\varphi_d - 1}} \tag{6}$$

where $\varphi_d = R_{b3}^d / R_{b3}$.

The value of the ultimate strength of concrete with indirect reinforcement ($R_{b3}$) in the first approximation according to studies [18] can be determined by the formula:

$$R_{b3} = \left[ \frac{1-\rho_{xy}}{2} + \left( \left( \frac{1-\rho_{xy}}{2} \right)^2 + 9\rho_{xy} \right)^{1/2} \right] R_b \tag{7}$$

where

$$\rho_{xy} = \psi_b \mu_{xy} \frac{R_s}{R_b} \tag{8}$$

In formulas (7) and (8), the following notation is used: $R_s$ and $R_b$ are, respectively, the calculated resistance of indirect reinforcement bars and the calculated resistance to compression of concrete; $\mu_{xy}$ - indirect reinforcement coefficient, defined by [19]; $\psi_b$ - the coefficient of uneven compression of the concrete core, depending on the shape of the cross section.

When calculating reinforced concrete elements of a constructive system for dynamic additions caused by a sudden change in power flows in this system, for example, a sudden change in the degree of static indeterminacy of the system, it is necessary to determine the values of dynamic stresses and, accordingly, dynamic curvatures in the considered sections of the system elements.

Suppose that in an arbitrary section of the *i* - th reinforced concrete element with indirect reinforcement of a statically indefinable system loaded n - times, the stress at an arbitrary point of the compressed concrete zone is equal to $\sigma_{b,n}^{st}$ (see Fig.1). This stress can be calculated by static calculation of the structural system at a given level of load. The stress $\sigma_{b,n-1}^{st}$ - is the stress in the same section, but *n-1* times a statically indefinable system if its transition from system n to system *n-1* would occur gradually, i.e. slow. With an instantaneous change in the static indeterminacy of a constructive system, a dynamic effect arises in its elements and, accordingly, the increment of stresses in the considered compressed zone of concrete reaching values on the $\sigma^{d}_{b,n-1}$ diagram. It is known that the maximum increment of these stresses in the $i$ - volume reinforced concrete element under consideration will occur at the first half-wave after the onset of oscillations of the reinforced concrete element.

As it was shown in [5,20] the potential energy level for an arbitrary $i$ - th reinforced concrete element, measuring it on the diagram relative to the static equilibrium point $\sigma^{st}_{b,n-1}$, is determined by the expression:

$$\Phi\left(\varepsilon_b\right) = \int_0^{\varepsilon_b} \sigma(\varepsilon_b) d\varepsilon_b \tag{9}$$

The condition of constancy of the specific energy of the $i$ - th reinforced concrete element leads to the following relation for the desired value of $\sigma^{d}_{b,n-1}$ or $\varepsilon^{st}_{b,n-1}$ (see Fig 1).

$$\Phi\left(\varepsilon^{d}_{n-1}\right) - \Phi\left(\varepsilon^{st}_{n}\right) = \sigma^{st}_{n-1}(\varepsilon^{d}_{n-1} - \varepsilon^{st}_{n}) \tag{10}$$

The analytical expression (9) means the equality of the areas of the curvilinear trapezium $\varepsilon^{st}_{b,n}$, *b*, *e*, $\varepsilon^{d}_{b,n-1}$ and the rectangle $\varepsilon^{st}_{b,n}$, *d*, *f*, $\varepsilon^{d}_{b,n-1}$, the common area for which is the area of the figure $\varepsilon^{st}_{b,n}$, *b*, *g*, *f*, $\varepsilon^{d}_{b,n-1}$.

In this way, the actual value of the strain $\varepsilon^{d}_{b,n-1}$ corresponds to the equality of the area of the triangles b, d, g and f, e, g.

For the considered linear portion of the concrete work diagram bc, with

$E^d_b = \dfrac{R^d_{b3} - \sigma^{st}_{bn}}{\varepsilon_{b33} - \varepsilon^{st}_{bn}}$, $\sigma^{st}_{bn} = \varepsilon E^d_b$ : (see Fig.1) and $\Phi(\varepsilon) = \dfrac{1}{2} E^d_b \varepsilon^2$.

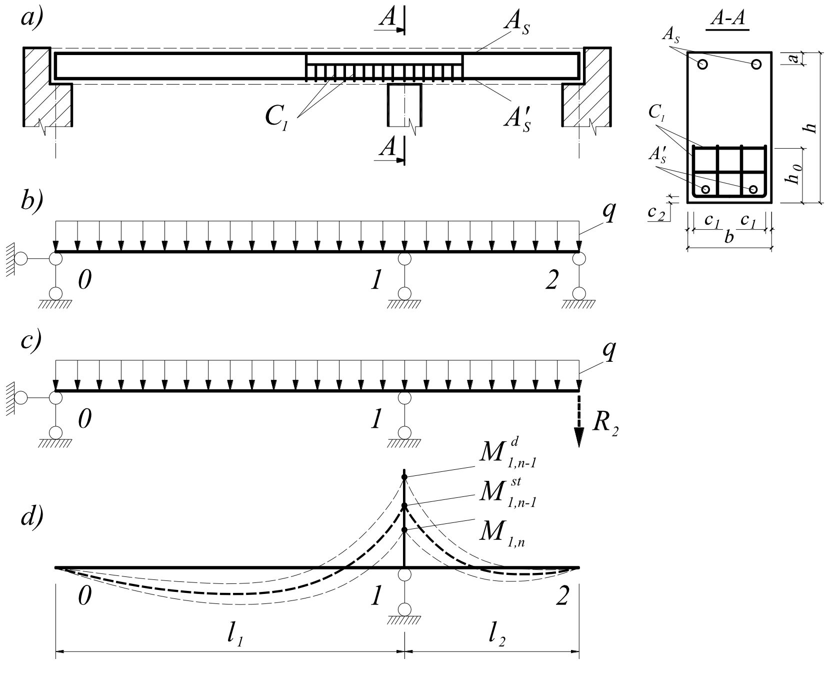


**Figure 2.** To the calculation of double-span reinforced concrete beams with double reinforcement and grids in the compressed zone in 70 mm increments.

Dependence (10) in stress can be written as:

$$\frac{1}{2}[(\sigma_{b,n-1}^{d})^{2}-(\sigma_{b,n}^{st})^{2}]=\sigma_{b,n-1}^{st}(\sigma_{b,n-1}^{d}-\sigma_{b,n}^{st}) \tag{11}$$

where does the expression:
$$\sigma_{n-1}^{d}=\sigma_{n-1}^{s}-\sigma_{n}^{s} \tag{12}$$

It is pertinent to note that the viscous element B of the mechanical model under consideration contributes to the inhibition of the development of deformations initiated in element A during the dynamic loading of a concrete element is essentially a damper of oscillations in it.

In experimental studies [18], it was found that under static loading of reinforced concrete bending elements by indirect reinforcement of the compressed zone, the reduction in the bearing capacity of the structure began at relatively large (about 1/20 span) deflections, i.e. The ultimate strain of reinforced compressed concrete $\varepsilon_{b33}$ significantly exceeds the deformation of unreinforced compressed concrete. In this regard, it can be assumed that indirect reinforcement of the compressed zone of the bent elements, other things being equal, leads to a decrease in the dynamic load factor of the structure equal to the ratio $\sigma_{b,n-1}^{d}$ to $\sigma_{b,n-1}^{st}$. And, accordingly, the risk of brittle fracture of the compressed zone of reinforced concrete bent elements is reduced.

To quantify this dynamic effect arising in a reinforced concrete structure of physically non-linear fragile material, consider the simplest numerical example. Let the plot of dynamic loading of compressed concrete on the "$\sigma_b-\varepsilon_b$" diagram is described by a nonlinear second-order dependence: $\varepsilon_b=(\sigma_b/B)^2$, where B is some constant.

Then, following (9) you can write:

$$\Phi(\varepsilon_b)=\int_0^{\varepsilon_b}\sigma(\varepsilon_b)\,d\varepsilon_b=\int_0^{\varepsilon_b}(\frac{\sigma_b}{B})^2\,d\varepsilon=\frac{2}{3}B\,\varepsilon^{3/2}=\frac{2}{3}\sigma_b^3/B^2 \tag{13}$$

or
$$\frac{2}{3}\,[(\sigma_{b,n-1}^{d})^{3}-\left(\sigma_{b,n}^{st}\right)^{3}\Big]=\sigma_{b,n-1}^{st}\left(\sigma_{b,n-1}^{d}-\sigma_{b,n}^{st}\right) \tag{14}$$

and dependence (10) is written in the form:

$$\frac{2}{3}[\left(\sigma_{b,n-1}^{d}\right)^{3}-(\sigma_{b,n}^{st})^{3}\Big]=\sigma_{n-1}^{st}(\sigma_{n-1}^{d}-\sigma_{n}^{st}) \tag{15}$$

Taking the ratio between stresses in compressed concrete in section A-A $\sigma_{b,n-1}^{d}/\sigma_{n}^{st}$ in the first approximation is the same as the relation between moments $M_{b,n-1}^{st}/M_{n}^{st}$ (see Fig.2), i.e. in the considered example being equal to 1.67 by solving equation (15) with respect to the desired stress $\sigma_{b,n-1}^{d}$ we get $\sigma_{b,n-1}^{d}$=1,29 $\sigma_{b,n-1}^{st}$.

In the case of linear dependence on the section of dynamic loading of compressed concrete "$\sigma_b-\varepsilon_b$", with the same ratio between $\sigma_{b,n-1}^{st}/\sigma_{n}^{st}$, we get:

$$\sigma_{b,(n-1)}^{d}=2\sigma_{n}^{st}=1,33\sigma_{n-1}^{st} \tag{16}$$

## 3. Numerical analysis

Using the obtained analytical dependencies, we determine the parameters of the dynamic expansion of compressed concrete in section A-A of a two-span reinforced concrete beam (see Fig.2, a) with a sudden removal of the extreme right support. The design diagram of the structure before and after removal of the support is shown in Fig.2, b, c, and the moment plot from the external static load is n-times the statically indefinable beam $M_{1n}^{st}$, in the beam n-1 times statically indefinable with the support $M_{n-1}^{st}$, and *n-1* times the statically indefinable beam $M_{n-1}^{d}$, taking into account the instantaneous applied support reaction $R_2$.

The initial data were taken for numerical analysis (see Fig.2, a): concrete B40, working overhead reinforcement 2Ø25 class A500, lower reinforcement 2Ø10 class A500. Beam section dimensions 250 × 160 mm. Characteristics of materials: $R_b$ = 22 MPa; $E_b$ = 36000 MPa; $\varepsilon_{bu}$ = 3.5 × $10^{-3}$; $R_s$ = 435 MPa; $A_s$ = 9.82 × $10^{-4}$ m2; $\varepsilon_{su}$ = 2.18 × $10^{-3}$; $R_{sc}$ = 435 MPa; $A_{sc}$ = 1.57 × $10^{-4}$ m2; $R_{s,xy}$ = 415 MPa; $\mu_{s,xy}$ = 0.026, $h_c$ = 70 mm; c = 30 mm; $c_1 = c_2$ = 15 mm; $a_s$ = 40 mm; $a_{sc}$ = 25 mm. The spans of the beam are respectively $l_1$ = 2000 mm, $l_2$ = 1000 mm. Distributed load q = 150 kN / m.

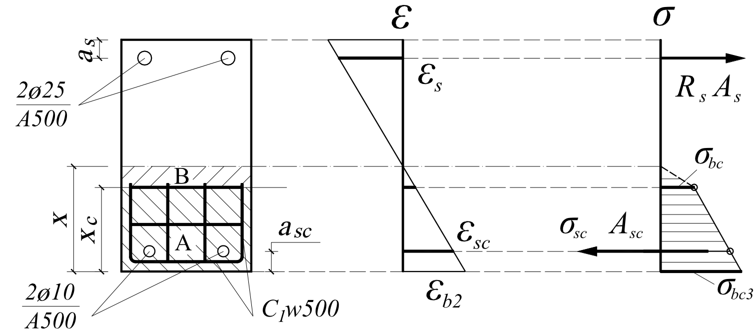


**Figure 3.** Scheme of the distribution of deformations (ε) and stresses ($\sigma$) along the height of the design section A-A.

In accordance with formulas (7) and (8) the values of the ultimate strength of concrete reinforced with grids:

$$R_{b3} = \left[\left(\frac{1-0{,}196}{2}\right)^2 + \left(\left(\frac{1-0{,}196}{2}\right)^2 + 9\times 0{,}196\right)^{1/2}\right]\times 22 = 34{,}11 MPa$$

The values of the dynamic strength of concrete $R_{b3}^d$ are determined by taking, according to [6], the value $\omega=\pi * 10^{-2}$ sec$^{-1}$ and the duration of the dynamic effect $t_d$ =$10^{-2}$ sec$^{-1}$ from equation (6) $\varphi_d$ =1,38 and, respectively, $R_{b3}^d$ =1,38*34,11=47.07 MPa.

Having determined the parameters of the static-dynamic diagram for the calculated section A-A by the values of limiting deformations on the most compressed face $\varepsilon_{b2}$ =3.5*$10^{-3}$ and $\sigma_s = R_s$, we determine the height of the compressed zone *x* = 13 cm.

The area of concrete A works with grids, and at section B without grids, we determine the stresses by height in compressed concrete and compressed reinforcement $\sigma_{b3}$ and $\sigma_{bc}$ and, accordingly, the forces in the compressed area of concrete and compressed reinforcement. From the calculated values of the stress, we obtain the limiting moment perceived by section A-A.

$$M_{ult} = \frac{1}{2}(\sigma_{bc3}+\sigma_{bc}) * b * x * \left(h_0 - \frac{x_c}{2}\right) + \frac{1}{3}\sigma_{bc} * b * (x - x_c) * \left(h_0 - \frac{4}{3}x_c - \frac{1}{3}x\right) + \sigma_{sc}A_{sc} = 70.15 \text{ kN*m}$$

Similarly, we calculate the maximum dynamic moment $M_{ult}^d$ taking into account when calculating the values of the maximum dynamic strength of concrete $R_{b3}$ and reinforcement $R_s^d$ , then the value $M_{ult}^d$ =99 kN*m.

## 4. Conclusions

The analysis of reinforced concrete statically indefinable structural system from progressive collapse by indirect reinforcement of the compressed zone of the bent elements showed that, all other things being equal, the bearing capacity of the sections with the dynamic loading of these elements can be increased to 29%. With a sudden redistribution of power flows in a structural system of reinforced concrete, indirect reinforcement plays the role of an additional damping element. This can be used in the development of methods to protect structures from progressive collapse.

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