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Backstepping Control of an Electro-Hydraulic Actuator using Kalman Filter

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***Abstract*—This paper presents the development of a Kalman filter-based controller for an electro-hydraulic actuator (EHA) system in the face of the disturbance white noise and unmeasurable system states. First, the continuous nonlinear EHA system is converted to a discrete system. Second, the discrete system is linearized to facilitate the filter design. Next, a Kalman filter (KF) is constructed to simultaneously estimate the unmeasurable system state and reduce the noisy sensor measurements. Then, backstepping control technique using KF approach is synthesized to not only obtain accuracy tracking control but also eliminate the disturbance white noise under unmeasurable system states. Finally, simulation evaluations and comparison studies are investigated to validate the feasibility of the presented method.**

Keywords—disturbance white noise, backstepping control, unmeasurable state, Kalman filter.

I. INTRODUCTION

The electro-hydraulic system has elicited much attention from scholars due to its advantages, for instance, fast response, reliability, high accuracy, and cost, etc. It became the potential selection for industry applications. Nevertheless, there are many negative factors such as highly complex nonlinear and noisy sensor measurements which make the tracking control problem challenge [1–3]. These issues affect not only the tracking performance but also system stability and should be tackled.

In order to reduce the impact of highly complex EHA nonlinear system, various control algorithms have been suggested such as PID, backstepping control, feedback linearization technique, adaptive control, and so on [4–8]. PID has been widely applied to other applications due to their simplicity [6, 7]. However, the PID controller cannot adapt for a large range of operating conditions in the presence of disturbances. In recent years, modern control has been investigated to improve tracking performance for the EHA system because it is a powerful and robust nonlinear strategy. They are good solutions to handle the complicated and inherited nonlinear characteristics of the EHA system. In [3], Du. Et al introduced a feedback linearization controller for a double rod EHA system to deal with the lumped disturbance and fault. In [8], adaptive control of hydraulic systems was proposed to achieve asymptotic tracking. In [9, 10], backstepping control is applied to the position control of an electrohydraulic servo system to guarantee the global asymptotic stability of the system. However, these approaches require the measurement of all system states. It increases the cost of the system and makes it more complex.

To deal with this challenge, Kalman filter is a potential tool which can observe the unmeasurable system state and suppress the noisy sensor measurements [11, 12]. Based on the above discussion, this paper suggests a KF based backstepping control to minimize the tracking error and ensure the system stability in the face of measurement noise and unmeasurable system states. Numerical simulations are put forward to make a comparison between traditional backstepping and KF based backstepping control under noisy sensor measurement.

The paper is organized as follows: In section II, the problem formulation is presented. Section III provides the control system design. The numerical results are described in section IV. The conclusion is given in section V.

II. PROBLEM FORMULATION

In this paper, the electro-hydraulic system contains rotary cylinder, relief valve, controllable valve, hydraulic pump, and displacement sensor which is described in Fig. 1. Defines the system sates $x=\begin{bmatrix}x_1 & x_2 & x_3 & x_4\end{bmatrix}=\begin{bmatrix}y_p & \dot{y}_p & P_1 & P_2\end{bmatrix}$, the fourth-order nonlinear EHA system is formulated as [2, 5]:

$$\begin{cases}\dot{x}_1 = x_2,\\ \dot{x}_2 = \dfrac{A}{m}\left(x_3 - x_4\right) - \dfrac{B}{m}x_2,\\ \dot{x}_3 = \dfrac{\beta_e}{V_1}\left(-Ax_2 - q_L + Q_1\right),\\ \dot{x}_4 = \dfrac{\beta_e}{V_2}\left(-Ax_2 + q_L - Q_2\right),\end{cases} \tag{1}$$

where y_p and $\dot{y}_p$ denote position and velocity of load, respectively; P_1 and P_2 are pressures inside the two chambers of the cylinder, respectively; $V_1 = V_{01} + Ay_p$ and $V_2 = V_{02} - Ay_p$ are the total control volume of the i^{th} (i = 1, 2) chamber, respectively; β_e is the bulk modulus of hydraulic oil; A, B, and m denote the radian displacement of the actuator, viscous friction coefficient, and mass of the load, respectively. The internal leakage of cylinder q_L can be calculated by [4]

$$q_L = c_L\left(P_1 - P_2\right) = c_L\left(x_3 - x_4\right), \tag{2}$$

where c_L is the internal leakage coefficient.

The supplied flowrate Q_i, i = 1, 2 go to forward and return chamber, which can be formulated as follows:

$$\begin{aligned} Q_1 &= k_t u\left[\zeta(u)\sqrt{P_s - x_3} + \zeta(-u)\sqrt{x_3 - P_r}\right],\\ Q_2 &= k_t u\left[\zeta(u)\sqrt{x_4 - P_r} + \zeta(-u)\sqrt{P_s - x_4}\right],\end{aligned} \tag{3}$$

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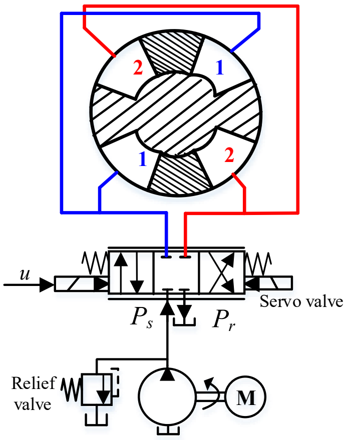

Fig. 1. EHA system model

where k_t is the gain of the control voltage u. P_s and P_r present the supply pressure and return pressure, respectively. The function $\zeta(u)$ can be described as follows:

$$\zeta(u)=\begin{cases}1 & if \quad u\geq 0,\\ 0 & if \quad u<0.\end{cases} \tag{4}$$

Assumption 1: Only the displacement is measured, and the desired trajectory is bounded.

In this paper, the control objective is to design a Kalman filter-based backstepping control for the system (1) such that the output $x_1(t)$ can exactly track the desired signal $x_{1d}(t)$ in the presence of disturbance white noise and unmeasurable states except for x_1.

III. CONTROL SYSTEM DESIGN

A. Kalman Filter

In this subsection, a Kalman filter is designed to not only estimate the unmeasurable system states but also eliminate the disturbance white noise. To facilitate the KF design, the continuous nonlinear EHA system is transferred to the discrete system. Next, the KF is constructed and its output supplies to the main controller. Finally, the KF based backstepping control approach is presented. The structure of the control approach is shown in Fig. 2.

Transferring the continuous domain to discontinuous domain, the below equations are applied as follows:

$$\begin{aligned}&x_n\to x_n(k),\\ &\dot{x}_n\to\frac{x_n(k+1)-x_n(k)}{T_s},\end{aligned} \tag{5}$$

where T_s is sample time,

A continuous nonlinear system (1) is converted to the discrete system as follows:

$$\begin{cases}x_1(k+1)=\phi_1\left(x_1(k),x_2(k)\right),\\ x_2(k+1)=\phi_2\left(x_2(k),x_3(k),x_4(k)\right),\\ x_3(k+1)=\phi_3\left(x_2(k),x_3(k),x_4(k),u(k)\right),\\ x_4(k+1)=\phi_4\left(x_2(k),x_3(k),x_4(k),u(k)\right),\end{cases} \tag{6}$$

where $\phi_1(.)=T_s x_2(k)+x_1(k)$,

$$\phi_2(.)=T_s\left[\frac{A}{m}\left(x_3(k)-x_4(k)\right)-\frac{B}{m}x_2(k)\right]+x_2(k)$$

$$=\left(-\frac{B}{m}T_s+1\right)x_2(k)+\frac{A}{m}T_s x_3(k)-\frac{A}{m}T_s x_4(k),$$

$$\phi_3(.)=-\frac{A\beta_e}{V_1}T_s x_2(k)-\left(\frac{\beta_e}{V_1}T_s c_L-1\right)x_3(k)$$
$$+\frac{\beta_e}{V_1}T_s Q_1(k)+\frac{\beta_e}{V_1}T_s c_L x_4(k),$$

$$\phi_4(.)=T_s\left[\frac{\beta_e}{V_2}\left(-Ax_2(k)+q_L(k)-Q_2(k)\right)\right]+x_4(k)$$

$$Q_1(k)=k_t u(k)\begin{bmatrix}\zeta(u(k))\sqrt{P_s-x_3(k)}\\ +\zeta(-u(k))\sqrt{x_3(k)-P_r}\end{bmatrix},$$

$$Q_2(k)=k_t u(k)\begin{bmatrix}\zeta(u(k))\sqrt{x_4(k)-P_r}\\ +\zeta(-u(k))\sqrt{P_s-x_4(k)}\end{bmatrix},$$

and $q_L(k)=c_L\left(x_3(k)-x_4(k)\right)$.

In case of the appearance of the disturbance noise, the eq. (6) can be linearized as follows:

$$\begin{aligned}&x(k+1)=\mathbf{A}x(k)+\mathbf{B}u(k)+\boldsymbol{\delta}(k)\\ &y(k)=\mathbf{D}x(k)+\psi(k)\end{aligned} \tag{7}$$

where $\psi(k)$ and $\delta(k)$ denote the measurement noise and system noise, respectively. **A**, **B**, and **D** are the system, input, and output matrix, respectively. They can be calculated as follows:

$$\mathbf{A}=\left.\begin{bmatrix}\frac{\partial\phi_1}{\partial x_1}&\frac{\partial\phi_1}{\partial x_2}&\frac{\partial\phi_1}{\partial x_3}&\frac{\partial\phi_1}{\partial x_4}\\ \frac{\partial\phi_2}{\partial x_1}&\frac{\partial\phi_2}{\partial x_2}&\frac{\partial\phi_2}{\partial x_3}&\frac{\partial\phi_2}{\partial x_4}\\ \frac{\partial\phi_3}{\partial x_1}&\frac{\partial\phi_3}{\partial x_2}&\frac{\partial\phi_3}{\partial x_3}&\frac{\partial\phi_3}{\partial x_4}\\ \frac{\partial\phi_4}{\partial x_1}&\frac{\partial\phi_4}{\partial x_2}&\frac{\partial\phi_4}{\partial x_3}&\frac{\partial\phi_4}{\partial x_4}\end{bmatrix}\right|_{(x_v,u_v)},\mathbf{B}=\left.\begin{bmatrix}\frac{\partial\phi_1}{\partial u}\\ \frac{\partial\phi_2}{\partial u}\\ \frac{\partial\phi_3}{\partial u}\\ \frac{\partial\phi_4}{\partial u}\end{bmatrix}\right|_{(x_v,u_v)} \tag{8}$$

$\mathbf{D}=\begin{bmatrix}1&0&0&0\end{bmatrix},(x_v,u_v)$ are operaing point.

Assumption 2: The distribution of measurement noise and system noise are the multivariate Gaussian with zero mean and its covariance will be denoted $E[\psi\psi^T]=\boldsymbol{P}$ and $E[\delta\delta^T]=\boldsymbol{M}$.

For the system (7), the Kalman filter can be established as follows [12, 13]:

$$\begin{aligned}&\hat{x}(k+1)=\mathbf{A}\hat{x}(k)+\mathbf{B}u(k)+\boldsymbol{\Gamma}\left(y(k+1)-\hat{y}(k+1)\right)\\ &\hat{y}(k)=\mathbf{D}\hat{x}(k)\end{aligned} \tag{9}$$

where $\hat{x}(k)$ and $\hat{y}(k)$ are the estimated state and output, respectively. $\boldsymbol{\Gamma}$ is the Kalman filter gain matrix. $\boldsymbol{\Gamma}$ can be computed by [12]

$$\boldsymbol{\Gamma}(k)=\mathbf{A\Omega}(k)\mathbf{D}^T\left(\mathbf{D\Omega}(k)\mathbf{D}^T+\mathbf{P}\right)^{-1} \tag{10}$$

where $\boldsymbol{\Omega}$ is the solution of the Riccati equation as follows:

$$\boldsymbol{\Omega}(k+1)=\mathbf{A\Omega}(k)\mathbf{A}^T+\mathbf{M}-\mathbf{A\Omega}(k)\mathbf{D}^T\mathbf{P}^{-1}\mathbf{D\Omega}(k)\mathbf{A}^T \tag{11}$$

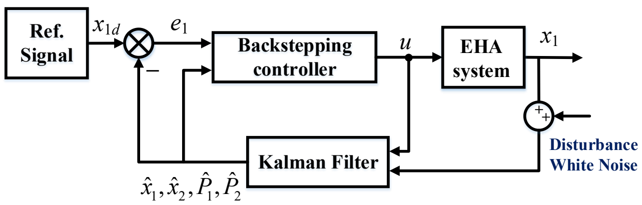

Fig. 2. Block diagram of the proposed FTC

B. Backstepping control design

Defines tracking errors as follows:

$$
\begin{aligned}
e_1 &= \hat{x}_1 - x_{1d}, \\
e_2 &= \hat{x}_2 - \alpha_1,
\end{aligned}
\tag{12}
$$

where x_{1d} is the desired signal, α_{i-1} denotes the virtual controller, which will be designed later.

Step 1: The Lyapunov function is selected as follows:

$$V_1 = \frac{1}{2}e_1^2 \tag{13}$$

The time derivative of V_1 is computed as follows:

$$\dot{V}_1 = e_1\left(\dot{\hat{x}}_1 - \dot{x}_{1d}\right) = e_1\left(\hat{x}_2 - \dot{x}_{1d}\right) \tag{14}$$

The virtual control law α_1 is constructed as

$$\alpha_1 = \dot{x}_{1d} - \kappa_1 e_1,\ \lambda_1 > 0 \tag{15}$$

Step 2: Considering the Lyapunov function as follows:

$$V_2 = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 \tag{16}$$

Differentiating V_2 with respect to time and noting (14), (15), one obtains

$$\dot{V}_2 = e_1\left(e_2 - \kappa_1 e_1\right) + e_2\left(\dot{\hat{x}}_2 - \dot{\alpha}_1\right) \tag{17}$$

The virtual control law α_2 is designed as follows:

$$\alpha_2 = -\kappa_2 e_2 - e_1 + \dot{\alpha}_1 + B\hat{x}_2/m,\ \kappa_2 > 0 \tag{18}$$

Step 3: Defines the new system state $x_{3a} = \dfrac{A}{m}\left(x_3 - x_4\right)$. Its derivative can be calculated as follows:

$$\dot{x}_{3a} = -f_2\gamma_2 - f_3\gamma_3 + \gamma_1 f_1 u \tag{19}$$

where $\gamma_1 = \beta_e k_t, \gamma_2 = \beta_e, \gamma_3 = \beta_e c_L, f_1 = \dfrac{A}{m}\left(\dfrac{Q_1}{V_1} + \dfrac{Q_2}{V_2}\right)$,

and $f_2 = \dfrac{A^2 x_2}{m}\left(\dfrac{1}{V_1} + \dfrac{1}{V_2}\right), f_3 = x_3\left(\dfrac{1}{V_1} + \dfrac{1}{V_2}\right)$.

Defines tracking error: $e_3 = \hat{x}_{3a} - \alpha_2, x_{3a} = \dfrac{A}{m}\left(\hat{x}_3 - \hat{x}_4\right)$.

The Lyapunov function be selected as

$$V_3 = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2}e_3^2 \tag{20}$$

Noting (17), (18), and (19), the time derivative of V_3 can be computed as follows:

$$\dot{V}_3 = -\kappa_1 e_1^2 - \kappa_2 e_2^2 + e_2 e_3 + e_3\left(\dot{\hat{x}}_{3a} - \alpha_2\right) \tag{21}$$

The final control signal u can be constructed as follows:

$$u = \frac{1}{\hat{f}_1\gamma_1}\left(-\kappa_3 e_3 + \hat{f}_2\gamma_2 + \hat{f}_3\gamma_3 - e_2\right),\ \kappa_3 > 0 \tag{22}$$

where $\hat{f}_1, \hat{f}_2$, and $\hat{f}_3$ are the nonlinear functions f_1, f_2, and f_3 using the estimated states of KF, respectively.

Theorem 1: For the EHA system provided by (1), with the presented KF in (9) and KF gain matrix in (10), if the virtual and actual control law are developed as (15), (18), and (22), all system states $e_i, i = 1,2,3$ are bounded in the small region.

IV. SIMULATION STUDY

A. Simulation setup

In order to evaluate the feasibility of the presented method, the simulation results are given in MATLAB2022a with the sampling time of 0.1 ms. The total simulation time is 10 s. The desired signal is $x_{1d} = 20\sin(\pi t)$ (°). The parameters of the EHA system are listed in TABLE 1 [5].

TABLE 1. PARAMETERS FOR THE EHA SYSTEM

Symbol	Value	Symbol	Value
m	0.37 kg	P_0	0.1 MPa
A	1.2×10^{-4} m³rad⁻¹	β_e	1.5×10^{9} bar
k_t	2×10^{-8} $\mathrm{m^3/s/V/Pa^{-1/2}}$	c_L	1.8×10^{-12} $\mathrm{m^3/s/Pa}$
P_s	10 MPa	B	45 $\mathrm{Nmsrad^{-1}}$
V_{01}	1.12×10^{-4} m³	V_{02}	1.12×10^{-4} m³

The control gains of the backstepping controller (BC) are selected by trial-and-error method as follows: $\kappa_1 = 150$, $\kappa_2 = 240$, $\kappa_3 = 90$. The operating point is set as $x_v = [1\ 0\ 3\times10^5\ 0]$, $u_v = 0$. The covariance matrices are selected as $\mathbf{M} = 0.001\times\mathrm{diag}([1\ 1\ 1\ 1])$, $\mathbf{P} = 10^{-5}$, $\mathbf{G} = \mathrm{diag}([1\ 1\ 1\ 1])$. The Kalman gain matrix can be computed through Matlab command $\mathbf{\Gamma} = lqe(\mathbf{A},\mathbf{G},\mathbf{D},\mathbf{M},\mathbf{P})$, which is shown as follows:

$$\mathbf{\Gamma} = \begin{bmatrix} 10.9549 & 0.0306 & -1.3781 & 8.2565\times10^{-7} \end{bmatrix}^T$$

The control system design in Matlab/Simulink is displayed in Fig. 3.

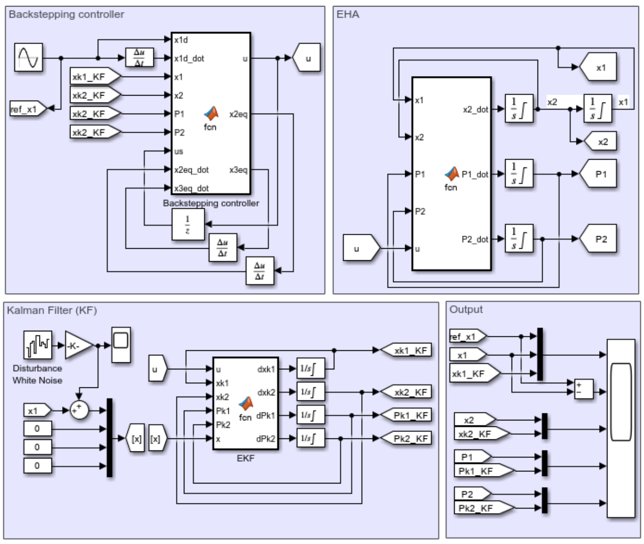

Fig. 3. Control system design in Matlab/Simulink

B. Simulation results.

To compare the simulation control results, the disturbance white noise is added as shown in Fig. 4. Two case studies are considered. Firstly, the BC is applied to the EHA system without using KF and all system states are measured. Secondly, the KF based BC is given and only output position is measured.

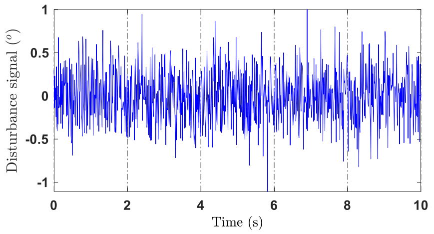

Fig. 4. Disturbance white noise

Case study 1:

The tracking for a sinusoidal reference signal is investigated by simulation. Fig. 5 exhibits the simulated output responses of the EHA system using the BC. We can see that the output position can track the desired signal. However, the disturbance noise negatively affects the tracking performance as displayed in the second subgraph in Fig. 5. The noise also appears in the other output system states x_2, P_1, P_2 as shown in Fig. 6.

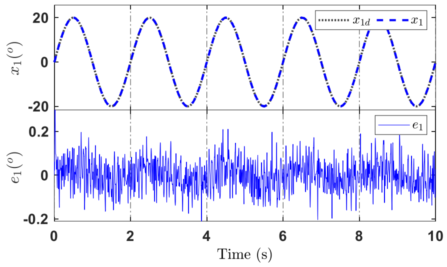

Fig. 5. Tracking performance of case study 1.

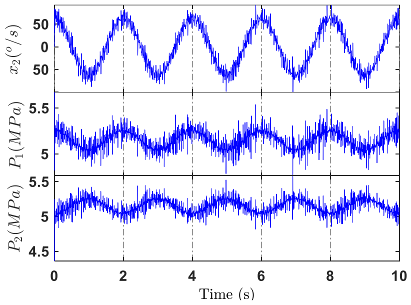

Fig. 6. Output system state x_2, P_1, P_2 in case study 1.

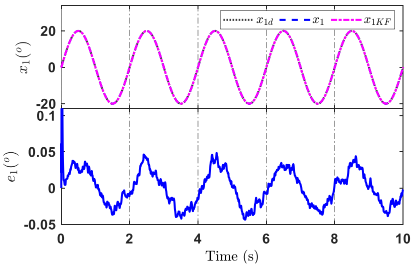

Fig. 7. Tracking performance of case study 2.

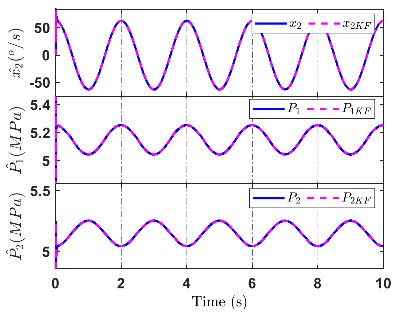

Fig 8. Output system state x_2, P_1, P_2 in case study 2.

Case study 2:

In this case study, KF based BC is used to improve tracking performance and eliminate the effect of disturbance noise. As seen in Fig. 7, the tracking error reduces from 0.2 to 0.05 when KF is applied. Furthermore, other unmeasurable states (i.e., x_2, P_1, P_2) can be estimated by KF as depicted in Fig. 8. It is noted that the disturbance noise is suppressed thanks to the designed KF.

V. CONCLUSION

This paper presented a KF based BC approach for the EHA system. The KF is constructed to reject the disturbance white noise and estimate the unmeasurable system state. The suggested method is performed to solve tracking control problems in the EHA system under noisy sensor measurements. The feasibility of the presented method was confirmed by the numerical simulation results. The physical data test will be investigated in future work.

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