Trajectory Tracking Control for a Variable Stiffness Robot 1-DOF including Pneumatic Actuator Dynamics

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Abstract— A variable stiffness actuator is a type of nextgeneration compliance control that allows the stiffness to be adjusted according to the designer's preferences. However, the tracking control issue of variable stiffness actuator is challenging due to their highly nonlinear behaviour. In this study, a terminal sliding mode control (TSMC) for the adjustment stiffness rotary actuator (ASRA) using pneumatic cylinder is proposed to improve tracking accuracy and robustness against uncertainties as well as disturbance. Here, the position and stiffness are independently controlled. Numerical simulations results are implemented to verify the effectiveness and capability of the proposed controller.

Keywords— Variable stiffness actuator (VSA), terminal sliding mode control, pneumatic cylinder, actuator dynamics

I. INTRODUCTION

The conventional rigid robot manipulator is widely used in many industrial applications. However, the joint stiffness of these robots remains constant, which presents limitations when used in specialized fields that require adjustable joint stiffness, such as entertainment robots, robots interacting with humans, rescue robots operating in confined spaces, and especially robots designed for children, where a soft, flexible, and huggable touch is desired [1], [2]. Consequently, research on variable stiffness actuator (VSA) robots has gained attention in recent years [3], [4]. These VSA robots feature two integrated components within each joint: the output link and the stiffness mechanism. The trajectory tracking control problem of VSA robots is challenging due to their highly nonlinear behavior. Therefore, studying how to ensure highprecision tracking performance for both components of VSA robots is essential.

The main innovations of this paper can be summarized as follows: 1. A terminal sliding mode controller is suggested for a variable stiffness robot with 1-DOF (VSR1D) using a pneumatic cylinder, designed to address both uncertainties, the impacts of the stiffness mechanism and to improve trajectory tracking accuracy. 2. The mathematical model of the VSR1D is developed, taking into account the dynamics of the pneumatic actuator.

The remainder of this paper is arranged as follow. In section II, the modelling of VSR1D including pneumatic actuator dynamics are formulated. Section III of this paper proposes a control strategies approach for VSA that deployed TSMC to cope with uncertainty and disturbances. Simulation results are carried out in section IV to demonstrate the effectiveness and capability of the suggested methodology. At the end of the paper, key areas for future work and the conclusion are discussed

II. PROBLEM FORMULATION

A. Dynamics system

A variable stiffness robot 1-DOF (VSR1D) with the integration of an ASRA using linear springs is displayed in Fig. 1. The dynamic system of VSR1D considering external load and gravity can be illustrated as follows [5]:

$$\begin{cases} J_{0}\theta + D_{0}\theta + \tau_{g} + \tau_{e} = \tau_{ext} \\ J_{1}\ddot{\theta}_{1} + D_{1}\dot{\theta}_{1} - \tau_{e} = \tau_{m1} \\ J_{2}\ddot{\theta}_{2} + D_{2}\dot{\theta}_{2} + \tau_{k} = \tau_{m2} \end{cases}$$
(1)

where θ , θ_1 , and θ_2 denote the angle of the output shaft, the positions of the pneumatic cylinder, and stiffness motor; J_i (*i*=0, 1, 2) defines the inertias of the output link, the inertias of pneumatic cylinder, and the inertias of stiffness part; τ_g is the gravity term; D_i presents the damping on the output link and relevant actuators; τ_{m1} and τ_{m2} denote the control input torque supplied by stiffness and pneumatic actuator.



Fig 1. Variable stiffness robot-1DOF

B. Pneumatic modelling

The model of the pneumatic servo system can be expressed by:

$$F = m\ddot{y} = P_{L} - F_{f}(y, \dot{y}, t) + d(y, \dot{y}, t)$$

$$P_{L} = A_{1}P_{1} - A_{2}P_{2}$$
(2)

where P_1 , P_2 , and P_L represent pressures in chambers A, B, and load pressure, respectively. A_1 and A_2 are the effective area inside chambers A and B, *m* is the total mass of the pneumatic cylinder, F_f denotes the friction force and *d* is disturbance.

The torque is applied to the output link position can be calculated as follows:

$$\tau_{m1} = J^T F = J^T \left[P_L + d\left(y, \dot{y}, t \right) \right]$$
(3)

where J defines the differentiable actuator Jacobian matrix, and $y = J\theta_1$.

So, the total system modelling of the variable stiffness robot 1-DOF can be expressed as follows:

$$\begin{aligned} \tau_{e} &= J_{1}\theta_{1} + D_{1}\theta_{1} - \tau_{m1} \\ \ddot{\theta} &= J_{0}^{-1} \left(\tau_{ext} - D_{0}\dot{\theta} - \tau_{g} - \tau_{e} \right) \\ &= J_{0}^{-1} \left(\tau_{ext} - D_{0}\dot{\theta} - \tau_{g} - J_{1}\ddot{\theta}_{1} - D_{1}\dot{\theta}_{1} + \tau_{m1} \right) \end{aligned}$$
(4)

III. CONTROLLER DESIGN

In this section the terminal sliding mode control technique is suggested to achieve the well tracking trajectory of the output position of variable stiffness robot 1-DOF. Meanwhile, the stiffness controller utilizes the conventional PID controller to regulate the stiffness of the robot 1-DOF.

Defining the tracking error: $e = \theta_d - \theta$, where θ_d denotes the desired output link position signal.

The sliding surface is chosen as follows:

$$s = \ddot{e} + a\dot{e} + be \tag{5}$$

where *a* and *b* denote the designed gain. The control law:

$$u = K\Phi_{\mu}(s) = Ksign(s)|s|^{\mu}$$
(6)

where *K* is the control gain, $\mu = a_1/b_1$, $a_1 > b_1$ being positive odd number.

IV. SIMULATION RESULTS

To verify the effectiveness and capability of the suggested methodology, the numerical simulation study is carried out in Matlab/Simulink as shown Fig. 2.



Fig. 2. Schematic diagram of the suggested method in Matlab/Simulink.

Two other controllers are given to compare with the proposed controller (a = 50, b = 500, K = 60, $\mu = 9/11$), i.e., Computed torque and PD control with feedforward compensation.

Fig. 3 shows the tracking performance of the output link position. As seen in Fig. 3, the curves of actual output link

position θ and θ_d are closely, however, the suggested control method exhibits the outperform tracking performance. Moreover, the tracking error *e* of the relevant controllers are displayed in Fig. 4, it implies that the tracking error of the suggested algorithm is the smallest as compared to two other controllers.



Fig 3. Output joint position tracking of the VSR1D



Fig 4. Error output joint position tracking of the VSR1D

V. CONCLUSION

In this paper, the trajectory tracking problem of VSR1D is presented. The modelling of VSR1D including pneumatic actuator dynamics is given. To ameliorate the tracking performance of the output position link, the terminal sliding mode controller is designed. The efficiency of the suggested methodology is demonstrated through the simulation results.

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